[redacted]

Hence, when A I and B I form an angle with each other so oblique as to make it difficult to determine precisely their point of intersection, we may proceed as follows to increase the precision of that determination:—Lay off any convenient equal distances, A d' = B d'', along A B from A and B respectively, to represent the longitudinal component of their velocities. Then complete the rectangular parallelograms A d' a f, B d'' b g; draw the straight line $f g$, cutting A B in D. Then from D perpendicular to A B draw D I; this line will traverse the instantaneous axis, and will increase the precision with which it is determined.

This last way of considering the motion of the piece is equivalent to regarding that motion as compounded of a rotation about an axis at D and a translation of that axis, and of the whole body along with it, with the velocity represented by D d.

III. *Given* (in fig. 37 or fig. 38), *the projections* A *and* B, *at a given instant, of two points in a moving piece on the plane of motion, and the ratio of their velocities, which are both perpendicular to the*

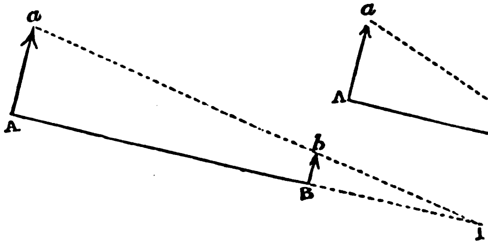

Fig. 37.

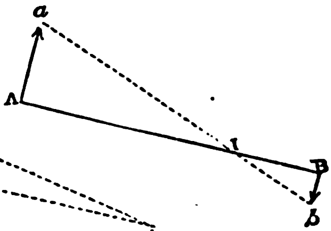

Fig. 38.

projection, A B, *of their line of connection, to find the* instantaneous axis *of motion of the piece.* Perpendicular to A B draw the straight lines A a, B b, bearing to each other the given proportion of the velocities of the two points: draw the straight line $a\,b$; the point of intersection, I, of A B and $a\,b$ (produced if necessary) will be the projection and trace on the plane of motion of the required instantaneous axis.

That axis may then be used as in the preceding Rule to determine the comparative motions of any set of points in the moving piece.

70. **Rotation about a Fixed Point.**—Every possible motion of a rigid body relatively to a point in the body is reducible to rotation about an axis, permanent, temporary, or instantaneous, as the case may be, which traverses that point. This is proved by showing that the following problem is always capable of solution:—

I. Given, at any instant, the directions of motion of any two points, B, C (fig. 39), in a rigid body relatively to a point, A, *in the*

[redacted]

of B and C; that is to say, of the two planes already mentioned, which traverse the instantaneous axis and the points B and C respectively; and I′ is *the trace of the instantaneous axis on the second plane of projection.* From I′ let fall I′ I perpendicular to B C; then I is the projection of I′ on the first plane of projection. Draw the straight lines A I, A′ I′: those are *the projections of the instantaneous axis.*

II. *To draw the projections of the points* B *and* C *on a plane perpendicular to the instantaneous axis, and to find the comparative motion of those points.* In B C, fig. 39, take I F = I′ A′; draw A G parallel and F G perpendicular to B C, cutting each other in G; join I G: this line will be the rabatment of I A. From B′ and C′ let fall B′ H and C′ K perpendicular to I′ A′ (produced if required). In B C take I L = I′ H, and I M = I′ K; then G, L, and M will represent the respective projections of A, B, and C upon a plane which traverses the instantaneous axis, and is perpendicular to the second plane of projection. From L and M let fall L N and M P perpendicular to I G. Then, in fig. 40, let the paper represent a plane of projection perpendicular to the instantaneous axis: let A be the trace and projection of that axis, and A l the trace of the plane already mentioned as being perpendicular to the second plane of projection in fig. 39. Make A l in fig. 40 = N L in fig. 39, and A m in fig. 40 = P M in fig. 39. Draw l B in fig. 40 perpendicular to A l in fig. 40 and = H B′ in fig. 39; also m C in fig. 40 perpendicular to A l in fig. 40 and = K C′ in fig. 39. Join A B, A C. Then B and C in fig. 40 will be *the projections required;* and the velocities of B and C relatively to A will be *perpendicular in direction and proportional in magnitude to* A B *and* A C *respectively.*

Another mode of finding the comparative motion of A and B is the following:—According to the principle of Article 54, page 32, the component velocities of B and C along their line of connection, B C, are equal. Therefore, in fig. 39, lay off along B C and B′ C′ the equal distances B d, C e, B′ d', C′ e', to represent that component; then draw d' b' d b, c' e' c e perpendicular to B C, cutting B b in b, B b' in b', C c in c, and C′ c' in c'; then B b and B′ b' will be the projections of the velocity of B relatively to A; and C e and C′ e' will be the projections of the velocity of C relatively to A. Then, by the rule of Article 19, page 7, find the lengths of the lines of which B b and B′ b', C c and C′ c' are the projections; the ratio of those lengths to each other will be the velocity-ratio of the two points.

71. **Unrestricted Motion of a Rigid Body.**—How complicated soever the motion of a rigid body may be, it may always be *regarded as made up* of a change of position of the body as a *whole—that is, a translation* of the body, and a change of position of

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sense, which comprehends cylinders and cones with bases of any figure, as well as those with circular bases.

In fig. 41, let the plane of the paper represent a plane of projection perpendicular to the straight line in which the fixed and the rolling surfaces touch each other; let T be the projection and trace of that straight line, which is the instantaneous axis of the rolling body. Let A be the projection at a given instant of a point in the rolling body; then at that instant A is moving with a velocity proportional to A T, and in a direction perpendicular to the plane traversing A and the instantaneous axis, of which plane A T is the trace.

It follows that the path traced by a point such as A in a rolling body is *a curve whose normal*, A T, *at any given point*, A, *passes through the corresponding position*, T, *of the instantaneous axis.* Curves of this class are called *rolled curves;* and some of them are useful in mechanism, as will be explained farther on.

73. **Composition of Rotation with Translation.**—From Article 52, page 30, it appears that the single rotation of a body about a fixed axis (such as O, fig. 19, page 26) may be regarded as compounded of a rotation with equal angular velocity about a moving axis parallel to the fixed axis (such as that whose trace is A, fig. 19), and a translation of that moving axis carrying the body along with it in a circle round the fixed axis of the radius O A. A similar resolution of motions may be applied to rotation about an instantaneous axis. For example, the rotation of the rolling body in fig. 41 about the instantaneous axis, T, may be conceived to be made up of a rotation about another axis, C, parallel to the instantaneous axis, and a translation of that axis.

The present Article relates to the converse process, in which there are given a rotation of a secondary piece about an axis occupying a fixed position in the piece, and a translation of that axis relatively to the frame in a direction perpendicular to itself—that is, parallel to the plane of rotation; and it is required to find, at any instant, the instantaneous axis so situated that a rotation about it with the same angular velocity shall express the resultant motion of the piece.

In fig. 41, let the plane of the paper be the plane of motion, and let C be the projection and trace of the moving axis—moving relatively to the frame, but fixed as to its position in the secondary piece. Let C U be the direction of the translation of that axis, carrying the moving piece with it; and let the velocity of translation be so related to the angular velocity of rotation as to be equal to the velocity of revolution about the axis C, of a particle whose *distance from* that axis is $\mathrm{C\,T} = \dfrac{\text{velocity of translation}}{\text{angular velocity}}$. Draw *C T of the length so* determined, in a direction perpendicular

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A MANUAL

OF

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PREFACE.

THIS book is divided into three parts: the first treats of the Geometry of Machinery; the second of the Dynamics of Machinery; and the third of the Materials, Strength, and Construction of Machinery.

Under the head of the Geometry of Machinery, machines are considered with reference to the comparative motions only of their moving parts; and rules are given for designing and arranging those parts so as to produce any given comparative motion.

Considering that the object of such rules is to adjust the dimensions of the parts of machines by processes of practical geometry, I have thought it advisable to solve every question by drawing, rather than by calculation, except in a few special cases where calculation is indispensable.

Many of the graphic rules thus obtained are made more easy and accurate, and some, indeed, are made possible which were not so before, by the aid of new methods of measuring and laying off the lengths of curved lines.

Two chapters of the first part are devoted to the detailed consideration of the movements of single pieces in machines. The remainder of the part relates to Pure Mechanism, as defined and reduced to a system by Professor WILLIS. The order in which the various combinations in mechanism are treated of is different from that adopted by him; but the principles are the same.

Several problems in mechanism are solved by methods which, so far as I know, have not hitherto been published; and which possess advantages in point of ease or of accuracy. I may specify, in particular, the drawing of rolling curves, and of some kinds of cams; the construction of the figures of teeth of skew-bevel wheels, and of threads of gearing screws, by the help of the normal section; and some improvements in the details of processes for designing intermittent gear, link-motions, and parallel motions.

Under the head of the Dynamics of Machinery are considered

[redacted]

Let T be the trace of the instantaneous axis, or line of contact of the cylinders, at the instant when the tracing point is at A; so that A T is the normal to the epicycloid at A, and O T and C T the radii of the fixed and rolling cylinders, being two parts of one straight line. Through O draw O E parallel to A C. Bisect O T in D, and draw the straight line A D E, cutting O E in E. Through E draw E F parallel to O T, and cutting A T (produced as far as required) in F. Then A F will be the radius of curvature of the epicycloid at the point A.

The following formula serves to find A F by calculation;

$$\mathrm{A\,F} = \frac{\mathrm{A\,T} \cdot \mathrm{O\,C}}{\mathrm{C\,D}}. \text{*} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

It is sometimes more convenient to calculate the distance, T F, of the *centre of curvature*, F, from the instantaneous axis, T, and that is done by the following formula:

$$\mathrm{T\,F} = \frac{\mathrm{A\,T} \cdot \mathrm{O\,D}}{\mathrm{C\,D}} = \frac{\mathrm{A\,T} \cdot \mathrm{O\,T}}{2\,\mathrm{C\,D}}; \ldots\ldots\ldots\ldots\ldots (2.)$$

the use of which, in designing the teeth of wheels by Mr. Willis's method, will appear farther on.

79. **To Draw Rolled Curves.**—A rolled curve may be drawn by actually rolling a disc of the form of the rolling curve, carrying a suitable tracing point, upon the edge of a disc of the form of the fixed curve. But it needs much care to perform that operation with accuracy, except with the aid of machinery specially contrived for the purpose, such as is to be found in certain kinds of turning lathes.

For ordinary purposes in designing machinery, approximate methods of drawing rolled curves are used, such as the following:—

I. *To draw approximately a rolled curve by the help of tangent circles.*—In fig. 47, let A B be the fixed curve, and A D the rolling curve, touching the fixed curve at A, which is also the position of the tracing point at starting. The curve A D rolls from A towards B; and it is required to draw approximately the curve traced by the point A. By Rule III. of Article 51, page 29, lay off on each of the two curves A B and A D a series of equal arcs, A 1, 12, 23, 34, &c. Measure the straight chord from 1 to A on the curve A D, and with 11 = 1A as a radius, and the point 1 on the curve A B as a centre, draw so much of a circle as lies near the probable position of the rolled curve; measure the straight chord from 2 to A on A D, and with 22 = 2 A as a radius, and the point 2 on the curve A B as a centre, draw in like manner part of a circle; and go on, in the same way, drawing a series of

* *The proof of this is as follows*:—Let the radius of the rolling cylinder, $CA = CT = r$; *let that of* the fixed cylinder, O T = R, which is to be

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free hand, or with the help of a bent spring, draw a curve, A E, so as to touch all those circular arcs; this will be very nearly the rolled curve required.

The curve A E is called the "Envelope" of the series of arcs that it touches.

II. *To find a series of points in a rolled curve.*—Draw a series of tangent circular arcs as in the preceding rule; then draw the several normals, 11, 22, 33, 44, &c., as radii of those arcs; the direction of each normal being determined by the principle, that at the point where it meets the fixed curve A B, it makes an angle with a tangent to that curve equal to the angle which the corre-

normal of the epicycloid, T A $= p$; and let the required radius of curvature, A F $= \rho$.

Let the angular velocity of the rolling cylinder, *relatively to the rotating plane* O C, be denoted by b, and that of the plane O C by a, so that the resultant angular velocity of the rolling cylinder is $a + b$. Then, because the angle C T A is the complement of one-half of the angle T C A, it is evident that the angular velocity of T A is $a + \frac{b}{2}$. But according to Article 76, $a \, R = b \, r$; therefore

$$a + b = b\left(1 + \frac{r}{R}\right); \; a + \frac{b}{2} = b\left(\frac{1}{2} + \frac{r}{R}\right).$$

In any indefinitely short time, $d\,t$, the normal sweeps through an angle whose value in circular measure is

$$d\,\theta = \left(a + \frac{b}{2}\right) d\,t = b\left(\frac{1}{2} + \frac{r}{R}\right) d\,t;$$

and the point A traces an arc of the length

$$d\,s = (a + b)\,p\,d\,t = b\left(1 + \frac{r}{R}\right) p\,d\,t;$$

therefore the radius of curvature of the epicycloid at the point A is

$$\rho = \frac{d\,s}{d\,\theta} = p \cdot \frac{1 + \frac{r}{R}}{\frac{1}{2} + \frac{r}{R}} = \frac{p\left(R + r\right)}{\frac{1}{2}R + r} = \frac{A\,T \cdot O\,C}{C\,D}.$$

This formula is made to comprehend the case of a cycloid by making $R = \infty$, when it becomes $\rho = 2\,p$; and that of the involute of a circle by making $r = \infty$, when we have $\rho = p$. When the epicycloid is internal, and R and r denote arithmetical values of those radii, the sign — is to be substituted for + both in the numerator and in the denominator of the formula. The symbolical expression for equation 2 of the text is

$$\rho - p = \frac{p\,R}{R + 2\,r},$$

with the same understanding as to the sign in the denominator. In the case already referred to at the end of Article 77, when a cylinder rolls inside a cylinder of twice its diameter, we have $R = -2\,r$, and the denominator of *the expression for* ρ becomes $= 0$; showing that the radius of curvature is *infinite; or, in other words*, that the epicycloid traced is a straight line, as *stated in the text.* When the rolling cylinder is concave, r is negative.

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points, A, D, B, C. Draw the diameter F E G, bisecting the arc A B in F and the arc B C A in G.

Draw the straight line G D, in which take G H = G A = G B. Through H, parallel to F E G, draw the straight line H K L, cutting A C in K and B C in L. Then about K, with the radius K A = K H, draw the circular arc A H; and about L, with the radius L H = L B, draw the circular arc H B: the curve made up of those two circular arcs will be a close approximation to the epicycloidal arc, having the same position and tangents at its two ends, and being very near to the true arc at all intermediate points.

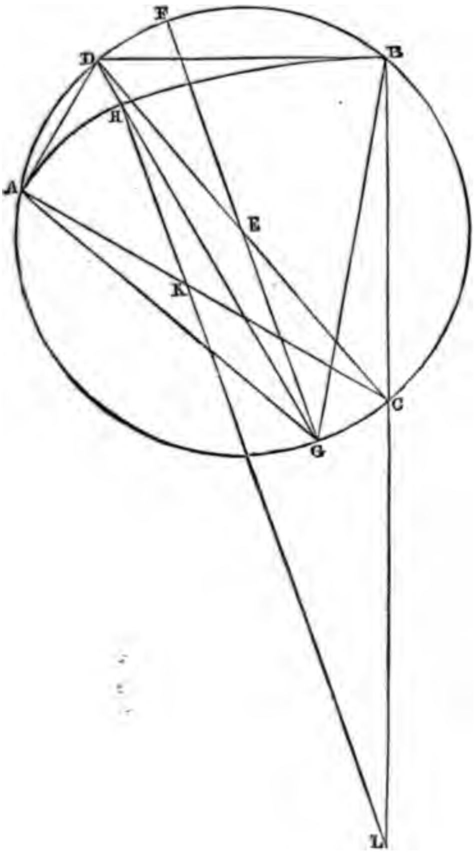

Fig. 48.

It may be remarked that G H = G A = G B = $\sqrt{(\mathrm{HK} \cdot \mathrm{HL})}$ approximates very closely to the mean radius of curvature of the epicycloidal arc A B; also that the process described is applicable to the approximate drawing of many curves besides epicycloids; and that the ratio of the two radii, H L : H K, deviates less from equality than that of the radii of any other pair of circular arcs which can be drawn so as to touch A D in A and B D in B, and also to touch each other at an intermediate point.*

* *This may be expressed symbolically by stating that* $\left(\frac{\mathrm{HL}^2 - \mathrm{HK}^2}{\mathrm{HK} \cdot \mathrm{HL}}\right)^2$ is a *minimum; or that* $\left(\log. \frac{HL}{HK}\right)^2$ is a minimum.

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equation then becomes of the *sixth order;* and it is to be solved so as to find h. This can be done by approximation only; and a convenient method of approximation is as follows:—Assume for q an approximate value, q', somewhat less than that of s (say $q' = 0{\cdot}9\ s$). Then calculate an approximate value, h', of the diameter, from equation (3), viz.:—

$$h' = \left(\frac{5{\cdot}1\ \mathrm{M}}{q'}\right)^{\frac{1}{3}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.)$$

Then calculate, for p, an approximate value, p', from equation (2), viz.:—

$$p' = (\cos i + m)\frac{4\ w\ c^2}{h'}; \ldots\ldots\ldots\ldots\ldots\ldots (6.)$$

and from the approximate value of p' calculate a second approximate value of q, as follows:—

$$q'' = \sqrt{(s^2 - s\ p')}. \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7.)$$

Should this agree with the first approximate value, q', the approximate diameter, h', will answer; and should there be a difference, a second approximation, h'', to the required diameter is to be computed, as follows:—

$$h'' = h'\left(\frac{q'}{q''}\right)^{\frac{1}{3}}. \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8.)$$

When, as is usually the case, the difference, $q' - q''$, is small compared with q', the following formula for the second approximation is sufficiently near the truth:—

$$h'' = h\left\{1 + \frac{q' - q''}{3\ q'}\right\}. \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9.)$$

A third approximation might be found by repeating the process; but the second approximation will, in general, be found accurate enough for practical purposes.

467. **Centrifugal Whirling of Shafts.***—Any small deflection of the centre line of a shaft from the straight axis of rotation gives rise, on the one hand, to centrifugal force, tending to make the deflection become greater; and, on the other hand, to elastic stress, resisting the deflection, and tending to straighten the centre line again. The resistance to deflection may be shortly called the *stiffness.* In very small deflections, the centrifugal force and the stiffness both increase according to the same law, being both sensibly propor-

* The *substance of this* Article first appeared in the Engineer of the *9th April, 1869.*

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portional to the area of a triangle having for its base the length marked on that axis, to represent that angular velocity, and for its summit the point E; so that the velocities of a particle at E due respectively to the rotations about

O A, O B, O C

are proportional respectively to the areas of the triangles

O A E, O B E, O C E.

Through A and B draw A G and B H perpendicular to O C, and join E G and E H. Then, by plane geometry,

$$OAE = OGE; \text{ and } OBE = OHE = GCE;$$

therefore

$$OCE = OGE + GCE = OAE + OBE.$$

So that the velocity of E due to the actual rotation about O C is the resultant of the velocities due to the rotations about O A and O B; the angular velocities being proportional to the lengths laid off on the axes respectively.

To prove the same proposition for a particle at D″, whose projection on the third plane is O, it is to be considered that the perpendicular distance of this point from the three axes, O A, O B, and O C, is identical, being the line marked O″ D″ and O′ D′ on the first and second planes; so that the velocities of D due to the three rotations are simply proportional to the three angular velocities. To represent, then, those three velocities as projected on the third plane, draw O *a*, O *b*, and O *c* perpendicular and proportional respectively to O A, O B, and O C. It is evident that O *a*, O *b*, and O *c* are the sides and diagonal of a parallelogram similar to O B C A; and therefore that the velocity of D″ due to the actual rotation about O C is the resultant of the velocities due to the rotations about O A and O B, the angular velocities being proportional to the lengths laid off on the axes respectively.

The proposition, therefore, is proved for both components of the velocity of a particle at F‴; and it holds for any particle whose projection on a plane perpendicular to the axis O C is F‴; that is, for every particle of the body, and therefore for the whole body: Q. E. D.

It appears, then, that rotations, when represented by lengths laid off on their axes proportional to their angular velocities, can be compounded and resolved, like linear velocities, by constructing *parallelograms.*

81. Rotations about Intersecting Axes Compounded. (*A. M.*, 392).

[redacted]

produce; and the "*Dynamics of Machinery*," which shows what modifications of force accompany given modifications of motion, and what modifications of motion are required in order to produce given modifications of force.

3. **Strength of Machinery.**—In order that a machine may be fit for use, every part, both of the machinery and of the framework, must be capable of bearing the utmost straining action which can be exerted upon it during the working of the machine, without any risk of being broken or overstrained; and the dimensions required for that purpose are to be determined by the proper application of the principles of the strength of materials.

4. The **Art of the Construction of Machinery** consists of three departments,—the selecting and obtaining of suitable materials for the parts of the mechanism and framework; the shaping of those parts to the proper figures and dimensions by means of suitable tools; and the fitting-up of the machine, by putting its parts together.

5. **Division of the Subject.**—For the reasons explained in the preceding Articles, the subjects of this work are treated of under four principal heads,—Geometry of Machinery, or Pure Mechanism; Dynamics of Machinery; Materials, Construction, and Strength of Machinery.

[redacted]

and, combining this proportion with that given in equation 1, we obtain the following proportional equation :—

$$\left.\begin{array}{lccccc} & \sin C O T & : & \sin A O T & : & \sin A O C \\ :: & a & : & b & : & c \\ :: & O a & : & O b & : & O c \end{array}\right\} \ldots\ldots(2.)$$

That is to say, *the angular velocities of the component and resultant rotations are each proportional to the sine of the angle between the axes of the other two;* and *the diagonal of the parallelogram* O b c a *represents both the direction of the instantaneous axis and the angular velocity about that axis.*

82. **Rolling Cones.** (*A. M.*, 393.)—All the lines which successively come into the position of instantaneous axis are situated in the surface of a cone described by the revolution of O T about O C; and all the positions of the instantaneous axis lie in the surface of a cone described by the revolution of O T about O A. Therefore the motion of the secondary piece is such as would be produced by the rolling of the former of those cones upon the latter. Circular sections of the two cones are sketched in perspective in fig. 51.

It is to be understood that either of the cones may become a flat disc, or may be hollow, and touched internally by the other. For example, should $\angle$ A O T become a right angle, the fixed cone would become a flat disc; and should $\angle$ A O T become obtuse, that cone would be hollow, and would be touched internally by the rolling cone; and similar changes may be made in the rolling cone.

The path described by a point in or attached to the rolling cone is a *spherical epitrochoid;* and if that point is in the surface of the rolling cone, that curve becomes a *spherical epicycloid.* It will be shown in the next chapter how to draw such curves—not exactly, but with a degree of accuracy sufficient for practical purposes.

83. **Resolution of Helical Motion.**—The resolution of helical or screw-like motion into rotation about an axis and translation along that axis has already been treated of in the last section of the preceding chapter. It remains to be shown how a helical motion may be regarded as compounded of two rotations about two axes which are in different planes.

In fig. 52, let the lower part of the figure represent a plane of projection, and O A and O B the projections upon that plane of two axes which are both parallel to it, but not in the same plane. Let the upper part of the figure represent a second plane of projection perpendicular to the first plane; and let F′ G′ be the *projection on that second* plane of the *common perpendicular* of *those two axes (Article 36,* page 14). Let a rigid body have a

X Z Z X is the vertical plane of projection, and X Y Y X the horizontal plane of projection; B is the vertical projection, and C the horizontal projection of the point A; and those two projections completely determine the position of the point A; for no other point can have the same pair of projections.

9. The **Axis of Projection** is the line X X, in which the two planes of projection cut each other.

10. **Rabatment.**—When the two projections of an object are shown in one drawing, it is convenient to represent to the mind that the following process has been performed:—Suppose that the vertical plane of projection is hinged to the horizontal plane at the axis X X, and that after the projection of the object on the vertical plane has been made, that plane is turned about that axis until it lies flat in the position X *z z* X, so as to be continuous with the horizontal plane: thus bringing down the projection B to *b*. This process is called the *rabatment* of the vertical plane upon the horizontal plane (to use a term borrowed from the French "*rabattement*" by Dr. Woolley). The two points C and *b* are in one straight line perpendicular to X X. The process of rabatment may be conceived also to be performed upon a plane in any position when a figure contained in that plane is shown in its true dimensions on one of the planes of projection.

11. **Projections of Lines.**—The projection of a line is a line containing the projections of all the points of the projected line. The projection of a straight line perpendicular to the plane of projection is a point; for example, the projection on the vertical plane, X Z Z X (fig. 1), of the straight line A B, perpendicular to that plane, is the point B. The projection of a straight line in any other position relatively to the plane of projection is a straight line. If the projected line is parallel to the plane of projection, its projection is parallel and equal to the projected line itself; thus the projection on the horizontal plane, X Y Y X, of the horizontal straight line A B, is the parallel and equal line C D. If the projected line is oblique to the plane of projection, the projection is shorter than the original line.

The projections, on the same plane, of parallel and equal straight lines are parallel and equal. The projections, on the same plane, of parallel lines bearing given proportions to each other are parallel lines bearing the same proportions to each other. When the plane of a plane curved line is perpendicular to a plane of projection, the projection of the curve on this plane is a straight line, being the intersection of the plane of the curve with the plane of projection. When the plane of the projected curve is parallel to a plane of projection, the projection of the curve on this plane is similar and equal to the original curve. In all other cases, it follows from the preservation of the proportions of a set of parallel

[redacted]

rotation about an intermediate axis, O′ G′, in the same plane, with an angular velocity represented by

$$\mathrm{O\,C} = \mathrm{O'\,C'} = \mathrm{F'\,A'} + \mathrm{G'\,B'};$$

and that axis of resultant rotation divides the distance F′ G′ in the following proportion:—

$$\mathrm{O'\,C'} : \mathrm{F'\,A'} : \mathrm{G'\,B'}$$
$$:: \mathrm{F'\,G'} : \mathrm{O'\,G'} : \mathrm{O'\,F'}.$$

To find the point O′ by graphic construction, draw F′ H′, parallel to A O and G′ H′ parallel to B O, cutting each other in H′; then through H′ draw H′ O′ C′ parallel to O C.

Moreover, the component rotations represented by O F and O G, about the axes F′ and G′, are of equal and opposite angular velocities; and therefore, according to Article 76, page 54, they are equivalent to a translation in the direction O C, with a velocity represented by the product O F · F G.

That translation being compounded with the resultant rotation represented by O C, gives finally, for the resultant motion of the body, *a helical motion about the axis whose projections are* O C *and* O C′.

The *pitch* of that helical motion, or advance per turn, is found by multiplying the rate of advance, O F, F′ G′, by the time of one revolution, $\frac{6{\cdot}2832}{\mathrm{O\,C}}$; and is therefore equal to the *circumference of a circle whose radius is* $\frac{\mathrm{O\,F} \cdot \mathrm{F'\,G'}}{\mathrm{O\,C}}$. Draw F′ K′ perpendicular to O B, and G′ K′ perpendicular to O A, cutting each other in K′ (which will be in the straight line H′ O′ C′). Then it is evident that F′ K′ G′ and C A O are similar triangles; and because D A = O F, we have the following proportion:—

$$\mathrm{O\,C} : \mathrm{O\,F} :: \mathrm{F'\,G'} : \mathrm{O'\,K'} = \frac{\mathrm{O\,F} \cdot \mathrm{F'\,G'}}{\mathrm{O\,C}};$$

Therefore *the pitch of the resultant helical motion is equal to the circumference of a circle whose radius is* O′ K′; and *the rate of advance may be represented by the product* O C · O′ K′.

84. **Rolling Hyperboloids.**—Conceive the straight line O C to represent an indefinitely long straight edge, rigidly fastened to the arm O′ F′, and sweeping along with that arm round the axis O A; *then* conceive the same straight line to be rigidly fastened to the *arm O′ G′, and to* sweep along with this arm round the axis O B. *Thus are generated* a pair of surfaces called Rolling Hyperboloids,

two planes of projection. The *rabatment* of the trace C is represented by *c*.

A straight line parallel to one of the planes of projection has only one trace, being the point where it cuts the other plane of projection.

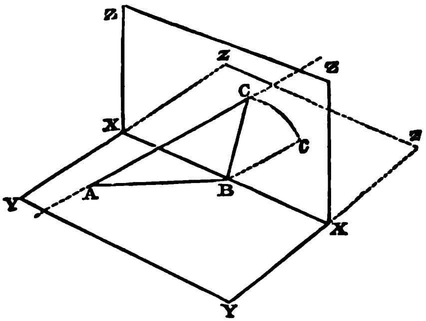

Fig. 2.

A straight line parallel to the axis of projection has no traces.

15. The **Traces of a Plane** are straight lines which (unless they are both parallel to the axis of projection) meet that axis in one point. The position of a plane is completely determined when its traces are known. For example, the plane A B C, in fig. 2, has its position completely determined by its traces, B A and B C.

A plane perpendicular to one of the planes of projection has its trace on the other plane of projection perpendicular to the axis of projection. A plane perpendicular to both planes of projection has for its traces two lines perpendicular to the axis. Thus, in fig. 1, page 3, the traces of the plane A B C D are D C and D B, both perpendicular to X X.

A plane parallel to one of the planes of projection has a trace on the other plane of projection only, being a straight line parallel to X X.

If a plane traverses a straight line, the traces of the plane traverse the traces of the line.

Section III.—*Rules Relating to Straight Lines.*

16. **General Explanations.**—In each of the figures illustrating the following rules the axis of projection is represented by X X; and in general the part of the figure above that line represents the rabatment of the vertical plane of projection, and the part below, the horizontal plane of projection. The projections of points on the horizontal plane are in general marked with capital letters, and the projections on the vertical plane with small letters.

17. **Given** (in fig. 3), **the Traces,** A, *b*, **of a Straight Line, to Draw its Projections.**—From A and *b* let fall A *a* and *b* B *perpendicular to X X.* Then *a* will be the vertical projection of the

[redacted]

cases proceed as follows:—Divide the circumference exactly, by plane geometry, into such a number of equal arcs as may be required in order to give sufficient precision to the approximative part of the process. Let the number of equal arcs in that preliminary division be called n. Divide one of them, by means of Rule V., into the required number of equal parts; n times one of those parts will be one of the required equal arcs into which the whole circumference is to be divided.

Rules I., III., and V., are applicable to arcs of other curves besides the circle, provided the changes of curvature in such arcs are small and gradual.

52. **Relative Translation of a Pair of Points in a Rotating Piece.**—In fig. 19, page 26 (where O, as already explained, is at once the projection and the trace of a fixed axis of rotation on a plane perpendicular to it, and A the projection of a point in the rotating piece), let B be the projection of another point in the rotating piece, and A B the projection of the straight line connecting those two points. The point B describes a circle of the radius O B about the fixed axis; and the radii O A and O B sweep round with the angular velocity common to all parts of the rotating piece, so that by the time that A has moved to the position A′, B has moved to the position B′, such that the angles A O A′ and B O B′ are equal. In order to determine the motion of one of those moving points (as A) relatively to the other (as B), it is to be considered that, owing to the rigidity of the body, the length of A B is invariable, and that the change of direction of that line (as projected on the plane of rotation), consists in turning in a given time through an angle equal to that through which the whole piece turns. In fig. 20, take B to represent at once the trace and the projection, on a plane of rotation, of an axis parallel to the fixed axis, and traversing the point B. Draw B A in fig. 20 parallel and equal to B A in fig. 19; and B A′ in fig. 20 parallel and equal to B′ A′ in fig. 19. Then A and A′ in fig. 20 represent two successive positions of A

with the summit pointing away from the centre of the arc; a straight line from the centre of the arc to that summit will bisect the arc. (2.) *To mark the sixth part of the circumference of a circle.* Lay off a chord equal to the radius. (3.) *To mark the tenth part of the circumference of a circle.* In fig. 24 A, draw the straight line A B = the radius of the circle; and perpendicular to A B, draw B C = ½ A B. Join A C, and from it cut off C D = C B. A D will be the chord of one-tenth part of the circumference of the circle. (4.) *For the fifteenth part,* take the difference between one-sixth and one-tenth. It may be added, that Gauss discovered a method of dividing the circumference of a circle by *geometry exactly,* when the number of equal parts is any prime number that *is equal to 1 + a power of 2*; such as $1 + 2^4 = 17$; $1 + 2^8 = 257$, &c.; but *the method is too laborious* for use in designing mechanism.

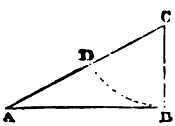

Fig. 24 A.

[redacted]

dicular to X X, cutting A B in G; A G will be the other projection of the given distance.

Another method of finding G is to lay off A G = *h f.*

21. **Given** (in fig. 4), **the Projections,** *a b,* A B, **of a Straight Line, to Find the Angle which it makes with One of the Planes of Projection** (for example, the horizontal plane).—Perform the construction described in Article 19; then *d e a* is the angle made by the given line with the horizontal plane. The same construction performed in the horizontal plane of projection will give the angle made by the given line with the vertical plane of projection.

22. **Given** (in fig. 5), **the Projections,** *a b* **and** A B, *a c* **and** A C, **of a Pair of Straight Lines which Intersect each other in the Point whose Projections are** *a,* A, **to find the Angle between those Lines.**—In either of the planes of projection (for example, the vertical

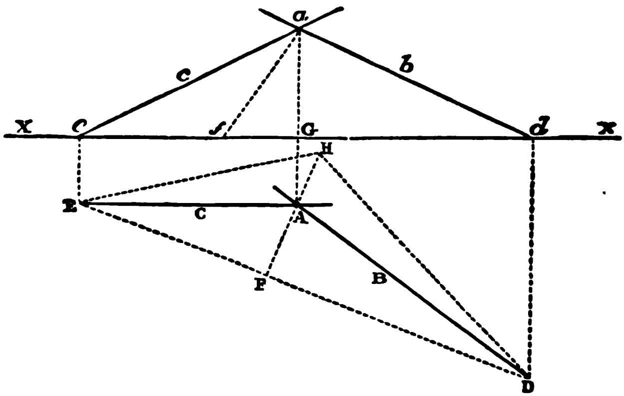

Fig. 5.

plane) find the points, *d, e,* where the projections of the given line cut the axis X X; these will be also the vertical projections of the horizontal traces of the lines. Through *e* and *d* draw *e* E, *d* D, perpendicular to X X, cutting A C and A B in E and D respectively; these points will be the horizontal traces of the lines. Join D E (which will be the horizontal trace of the plane containing the lines), and on it let fall the perpendicular F A. Join A *a* (which of course is perpendicular to X X); let it cut X X in G. Make G *f* = A F, and join *a f.* In F A produced, take F H = *a f*; join H E, H D; E H D will be the angle required.

Remark.—The triangle E H D is the *rabatment* upon the horizontal plane of the triangle whose projections are E A D and *e a d.*

[redacted]

the perpendicular distances of those points from the axis of rotation.

It is obvious that all points in a circular cylindrical surface described about the axis of rotation have equal velocities. The dotted circles in fig. 19, page 26, represent the traces of two such surfaces.

The relative motions of any two pairs of points in a rotating piece may be compared together. For example, let it be proposed to compare the motion of A relatively to B with the motion of B relatively to O. Then, because the velocity of the motion of A relatively to B is proportional to B A, and its direction perpendicular to the plane whose trace is B A, while the velocity of the motion of B relatively to O is proportional to O B, and its direction perpendicular to the plane of which O B is the trace, the directional relation is expressed by the angle made by those planes with each other, and the velocity-ratio by the ratio B A : O B borne to each other by the projections on the plane of rotation of the two lines of connection of the two pairs of points.

54. **Relative and Comparative Translation of a Pair of Rigidly Connected Points.**—The following proposition is applicable to all motions whatsoever of a pair of points so connected that the distance between them is invariable. It forms the basis of nearly the whole theory of combinations in mechanism, and many of its consequences will be explained in the ensuing chapters of this Part. At present it is introduced with a view to its application to pairs of points in a rotating piece.

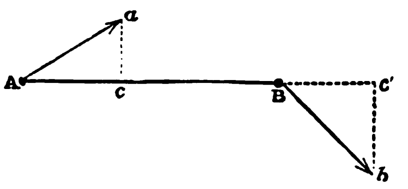

Fig. 25.

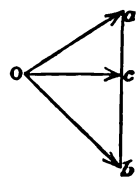

Fig. 26.

THEOREM.—*If two points are so connected that their distance apart is invariable, the components of their velocities along the straight line which traverses them both must be equal;* for if those component velocities are unequal, the distance between the points must necessarily change.

The straight line which traverses the points is called their *Line of Connection.*

For example, in fig. 25, let A and B represent two points in the plane of the paper, whose distance apart, A B, is invariable. At a *given instant* let the velocities of those points be represented by *straight lines, which may* be in the same plane, or in different planes,

[redacted]

example, B A) take any convenient point, A, from which let fall A D perpendicular to X X; and on B D as a diameter describe a circle. From D let fall perpendiculars, D *e*, D F, on the two given traces. From the point *e*, thus found on the opposite trace to that on which the point A was assumed, let fall *e* E perpendicular to X X; join E A, cutting D F in G. From G draw G H perpendicular to X X, cutting the circle in H; D B H will be the required angle.

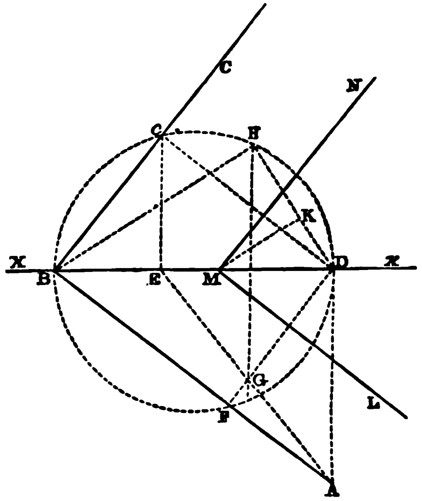

Fig. 7.

26. **Given (in fig. 7), the Traces of a Plane, B A, B C, to Draw the Traces of another Plane which shall be Parallel to the given Plane, and at a given Perpendicular Distance from it in either Direction.**—Complete the construction described in Article 25. Join D H (this represents the perpendicular distance of the point D in the axis from the given plane); then from H, along H D (or along D H produced, according to the direction in which the new plane is to lie), lay off the given perpendicular distance between the planes, H K. From K draw K M parallel to H B, cutting X X in M. From M draw M N parallel to B C, and M L parallel to B A; these will be the traces of the plane required.

Or otherwise:—Complete the construction described in Article 24 (see fig. 8). A *f* is the rabatment of the intersection of the given plane with a plane, A D *e*, perpendicular to the vertical trace B C. Through A draw A M perpendicular to A *f*, and make A M equal to the given distance between the planes; draw M N parallel to A *f*, cutting X X in N In D *e* produced take D O equal to D N. O is a point in the trace of the plane required. Through O draw O P parallel to B C, cutting X X in P; and through P draw P Q parallel to B A. O P Q is the plane required.

27. **Given (in fig. 9), the Traces of Two Planes, C A *d* and C B *d*, to Draw the Projections of their Line of Intersection.**—The traces of

[redacted]

let fall V U perpendicular to B A; then A U represents the component in question. Sometimes the more convenient way of finding that component is the following:—

From O let fall O B perpendicular to B A. Then A and B represent a pair of rigidly connected points; therefore, according to Article 24, the component velocities of A and B along A B are equal. But B A, being perpendicular to O B, is the direction of the whole velocity of B; therefore *the component, along a given straight line in the plane of rotation, of the velocity of any point whose projection is in that line, is equal to the whole velocity of the point where a perpendicular from the axis meets that line.*

The whole velocity of B is = O B × the angular velocity; and the velocity-ratio of B to A, or, in other words, the ratio of the component velocity of A along B A to the whole velocity of A, is O B : O A.

The velocity of a point such as A in a rotating piece may be resolved into components, oblique (see fig. 19) or rectangular (see fig. 27) as the case may be, by regarding the velocity of A relatively to O as the resultant of the velocity of A relatively to B, and of that of B relatively to O. The directions of that resultant velocity and its two components are respectively perpendicular to O A, B A, and O B, and their ratios to each other are equal to those of the lengths of the same three lines. This is a particular case of a more general proposition, viz.,—that *the velocities of three points relatively to each other are proportional to the three sides of a triangle which make with each other the same angles that the directions of those three relative velocities do* (*A. M.*, 355).

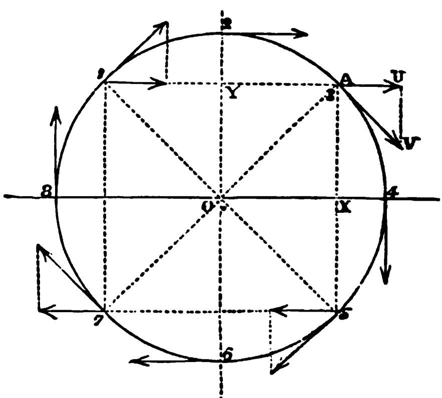

Fig. 28.

In fig. 28, let O be the trace of the axis on a plane of rotation, and A a point in the rotating piece, revolving in the circle O A, so as to assume successively a series of positions such as 1, 2, 3, 4, 5, 6, 7, 8; and in each position of A, let the component velocity *A U, parallel to* a fixed plane whose trace is the diameter 8 O 4, *be compared with* the whole velocity of revolution, A V.

[redacted]

to Find the Angle between them.—From either of the intersections of the traces (say d) let fall d D perpendicular to X X; draw D C, joining D with the other intersection of the traces. Through any convenient point, I, in D C, draw G I H perpendicular to D C,

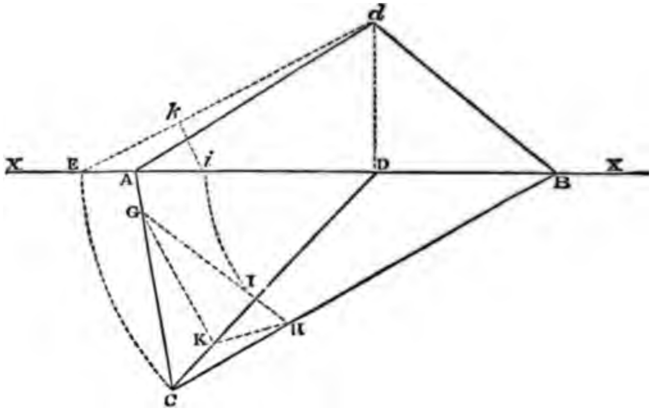

Fig. 10.

cutting A C in G and B C in H. Along X X lay off D E = D C, and D i = D I; join d E (this will be the length of the line of intersection of the planes). From i let fall $i\,k$ perpendicular to d E; in I C take I K = $i\,k$; join K G, K H; G K H will be the angle required.

When the traces of the two given planes are inconveniently placed for the completion of the figure, we may substitute for either pair of traces another pair of traces parallel to them, and more conveniently placed.

30. **Given** (in fig. 10), **the Traces,** A d **and** A C, **of a Plane; also the Traces,** d **and** C, **of a Straight Line in that Plane; to Draw the Traces of a Plane which shall Cut the given Plane in that Line at a given Angle.**—From either of the traces of the straight line, as d, let fall d D perpendicular to X X; draw the straight line D C, joining D with the other trace, C, of the straight line. Through any convenient point, I, in D C, draw I G perpendicular to D C, cutting C A in G. In X X lay off D E = D C and D i = D I; join d E, and on it let fall the perpendicular $i\,k$. In I C take I K = $i\,k$; join K G. Then draw K H, making G K H = the given angle, and cutting G I, produced if necessary, in H. Draw C H, cutting X X in B, and join B d; these will be the traces of the plane required.

[redacted]

jointly. Hence, let a and a' denote the angular velocities of two rotating pieces, or a pair of numbers proportional to those angular velocities; r and r', the perpendicular distances of a pair of points in those two pieces from their respective axes, or a pair of numbers proportional to those distances; and v and v', the respective velocities of those two points, or a pair of numbers proportional to those velocities; then the velocity-ratio of the points is,

$$\frac{v'}{v} = \frac{a' r'}{a r}.$$

In order that a pair of points in a pair of rotating pieces may have equal velocities—that is, in order that $\frac{v'}{v}$ may be $= 1$, we must make the radii inversely proportional to the angular velocities—that is, $a' r' = a r$, or $\frac{r'}{r} = \frac{a}{a'}$.

SECTION IV.—*Screw-like Motion of Primary Pieces.*

57. **Helical or Screw-like Motion** (*A. M.*, 382,) is compounded of rotation about a fixed axis, and of translation along that axis: the *advance* (as the translation in a given time is called) bearing a constant proportion to the rotation in the same time; in other words, the moving piece advances along the axis of rotation through an uniform length during each turn.

The subject of the resolution of screw-like motion into components in other and more complex ways will be considered in the next chapter.

58. **General Figure of a Screw—Pitch.** (*A. M.*, 471.)—In order that a primary moving piece may have screw-like motion, its figure ought to be that of a true screw; and it ought to turn in a bearing of the same figure, fitting it accurately. The figure of a screw may be described in general terms as consisting of a projection of uniform cross-section called the *thread*, winding in successive coils round a circular cylinder. The best form of section for the thread of a screw that is to be used as a primary moving piece for producing helical motion only, and not as a fastening, nor in "screw gearing," is rectangular. The forms suited for other purposes will be considered later. There are two sorts of screws, convex, or *external*, and concave, or *internal;* in the former the thread winds round the outside of a cylindrical spindle; in the latter it winds round the inside of a hollow cylinder. When the word "screw" is used without qualification, an external screw is usually meant; an internal screw is called a "*nut*." When a primary moving piece is an external screw, its bearing is an internal screw; when the primary moving piece is an internal screw, the bearing is an external screw. The truth or accuracy of

[redacted]

X X; from F draw F f perpendicular to X X, cutting the trace B C in f; join f e; from d draw d g perpendicular to B C, cutting f e in g; and from g draw g G perpendicular to X X, cutting D F in G. D G and d g will be the projections of the perpendicular required.

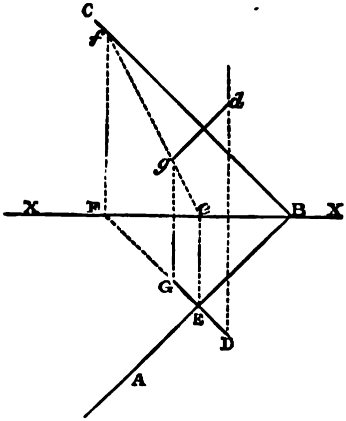

Fig. 12.

34. **Given (in fig. 13), the Projections of a Point, D, d, and those of a Straight Line, A B, a b, to Draw the Traces of a Plane which shall Traverse the Point, and be Perpendicular to the Line.**—Through one of the projections of the given point (say D) draw D G perpendicular to A B (the corresponding projection of the given line), cutting X X in G. Through G draw G g perpendicular to X X; through d, the other projection of the point, draw d g parallel to X X, cutting G g in g; through g draw E C perpendicular to a b, cutting X X in C; and through C draw C F perpendicular to A B. E C F will be the traces of the required plane.

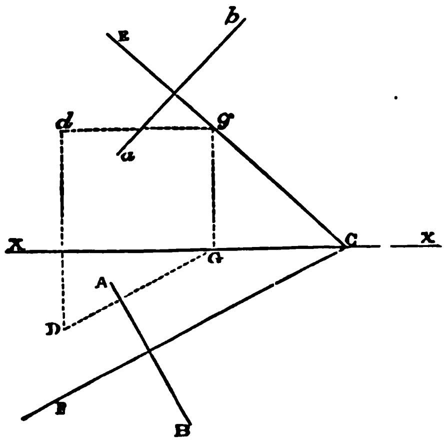

Fig. 13.

35. **Given, the Projections of a Point and of a Straight Line, to Draw the Projections of a Perpendicular let Fall from the Point upon the Straight Line.**—Find by the preceding rule the traces of a plane traversing the given point, and perpendicular to the given line; then, by Article 23, find the traces of a plane traversing the given point and line; and finally, by Article 27, find the projection of the line of intersection of those two planes.

36. **Given, the Projections of Two Straight Lines that are neither**

61. **Path of a Point in a Screw—Linear Screw or Helix.**—A point in, or rigidly attached to, a screw, traces a path which may be called a screw-shaped line or *linear screw*. By mathematicians it is called a *helix*. A helix winds round in successive similar coils upon a cylindrical surface described about the axis of rotation with a radius equal to the perpendicular distance of the tracing point from the axis. The distance, measured parallel to the axis, between any two successive coils is everywhere the same, and is identical with the *pitch* of the screw; and the angle of inclination of the linear screw to the axis is everywhere the same.

Points in, or rigidly attached to, a screw, at equal distances from the axis, trace by their motion equal and similar linear screws on one and the same cylindrical surface. Points at unequal distances from the axis trace different linear screws, inclined to the axis at different angles, and situated on cylindrical surfaces of unequal radii; but the *pitch* of all those linear screws is the same. All the edges, whether projecting or re-entering, of a screw-thread are linear screws.

A linear screw may be traced on a cylindrical surface by any mechanical contrivance which ensures that, while the cylinder rotates, the tracing point shall advance along a line parallel to the axis at a rate bearing a constant proportion to the rate of rotation. This will be further considered in that part of this treatise which relates to the construction of machinery.

A linear screw is the shortest line on the surface of a cylinder between two points that are neither in one plane traversing the axis nor in one plane perpendicular to the axis; and a cord or a flexible wire stretched on a cylindrical surface between two such points tends to assume of itself the figure of a linear screw.

62. **Projection of a Linear Screw.**—The most useful projection of a linear screw is that upon a plane traversing the axis, and is drawn as follows:—In fig. 30, let A B represent the axis of the screw. Draw D A C perpendicular to A B, making A C = A D = the radius of the cylindrical surface in which the helix is to be situated. Draw D I and C F parallel to A B; those two lines will be the traces of the cylindrical surface. About A, with the radius A C, draw the semicircle C K D; this represents the trace of one-half of the cylindrical surface on a plane perpendicular to its axis, "rabatted" upon the plane of projection. Divide the semicircle into any convenient number of equal arcs (Article 51, page 27); the greater the number of those divisions, the greater will be the accuracy of the projection. In fig. 29 the semicircle is divided into six equal arcs only; in practice a greater number will in general be required.

On C F, or any other line parallel to the axis, lay off C E = the intended pitch of the screw, and divide it into twice as many

L *l*, perpendicular to X X; and the diameters, *l m*, L M, are the projections of one diameter of the circle—viz., that diameter in which the plane A B C is cut at right angles by a plane parallel to X X. The perpendicular distance, N *n*, between the two tangents is equal to the diameter of the circle multiplied by the cosine of the angle which the given plane makes with X X, and is bisected by the line D *d*.

primary moving pieces in rolling contact, both may rotate, or one may rotate and the other have a motion of straight sliding. A rotating piece, in rolling contact, is called a *toothless wheel*, and sometimes a *roller;* a sliding piece may be called a *toothless rack*.

97. **Ideal Pitch Surfaces.**—The designing of pitch surfaces is used not only with a view to the making of toothless wheels and toothless racks (which are seldom employed), but much oftener as a step towards determining the proper figures for wheels and racks provided with teeth.

The pitch surface of a toothed wheel or of a toothed rack is an ideal smooth surface, intermediate between the crests of the teeth and the bottoms of the spaces between them, which, by rolling contact with the pitch surface of another wheel, would communicate the same velocity-ratio that the teeth communicate by their sliding contact. In designing toothed wheels and racks the forms of the ideal pitch surfaces are first determined, and from them are deduced the forms of the teeth.

Wheels with cylindrical pitch surfaces are called *spur wheels;* those with conical pitch surfaces, *bevel wheels;* and those with hyperboloidal pitch surfaces, *skew-bevel wheels.*

98. The **Pitch Line** of a wheel, or, in circular wheels, the PITCH CIRCLE, is the trace of the pitch surface upon a surface perpendicular to it and to the axis; that is, in spur wheels, upon a plane perpendicular to the axis; in bevel wheels, upon a sphere described about the apex of the conical pitch surface; and in skew-bevel wheels, upon an oblate spheroid generated by the rotation of an ellipse whose foci are the same with those of the hyperbola that generates the pitch surface. The pitch line might be otherwise defined, in most cases which occur in practice, simply as the trace of the pitch surface upon a plane perpendicular to the axis of rotation.

The PITCH POINT of a pair of wheels is the point of contact of their pitch lines; that is, the trace of the line of contact upon the surface or surfaces on which the pitch lines are traced.

The *pitch line of a rack* is the trace of its pitch surface on a plane parallel to its direction of motion, and containing its line of connection with the wheel with which it works.

A SECTOR is a name given to a wheel whose pitch-line forms only part of a circumference: sectors are used where the motion required is reciprocating or "rocking," and does not extend to a complete revolution. Everything stated respecting the figures of complete wheels applies also to the figures of sectors.

99. **General Conditions of Perfect Rolling Contact.** (*A. M.*, 439.) —The whole of the principles which regulate the motions of a pair *of primary pieces in* perfect rolling contact follow from the single *principle, that each pair of points in the pitch surfaces which are in*

[redacted]

surfaces, *exactly straight* in the direction of motion. The bearings of rotating pieces must have surfaces accurately turned to *figures of revolution*, such as circular cylinders, spheres, cones, conoids, and flat discs. The bearing of a piece whose motion is helical, must be an exact screw. Those parts of moving pieces which touch the bearings should have surfaces accurately fitting those of the bearings. They may be distinguished into *slides*, for pieces which move in straight lines, *gudgeons*, *journals*, *bushes*, and *pivots*, for those which rotate, and *screws* for those which move helically.

The accurate formation and fitting of bearing surfaces is of primary importance to the correct and efficient working of machines.

39. **The Motions of Primary Moving Pieces** (*A M.*, 429,) **are Limited** by the fact, that in order that different portions of a pair of bearing surfaces may accurately fit each other during their relative motion, those surfaces must be either straight, circular, or helical; from which it follows, that the motions in question can be of three kinds only, viz.:—

I. *Straight translation*, or *shifting*, which is necessarily of limited extent, and which, if the motion of the machine is of indefinite duration, must be *reciprocating;* that is to say, must take place alternately in opposite directions: for example, the piston-rod of a steam engine.

II. *Simple rotation*, or *turning* about a fixed axis, which motion may be either continuous or reciprocating, being called in the latter case *swinging*, *rocking*, or *oscillation*. Continuous rotation is exemplified by the shaft of a steam engine; reciprocating rotation by various beams or levers.

III. *Helical* or *screw-like motion*, compounded of rotation about a fixed axis, and translation along that axis.

SECTION II.—*Straight Motion of Primary Pieces.*

40. **Straight Translation** is the motion of a primary piece sliding along a straight guiding surface. All the particles of the piece move through equal distances in a given time, along parallel straight lines; and the line joining any two particles remains unaltered in length and in direction.

41. **Resolution and Composition of Motions.**—The *resultant* of two or more *component* motions is the motion which results from putting them together. If the component motions are represented by straight lines, their resultant is found geometrically by joining together, end to end, a series of straight lines respectively equal and parallel to the given straight lines, and pointing in the same directions, and then drawing a straight line from the starting point to the further end of the series. For example:—

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proportional to the intended angular velocities. Draw the straight line $a\ b$, cutting A B (produced if necessary) in K. Lay off B I =

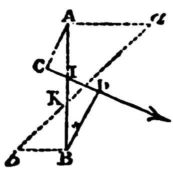
Fig. 59.

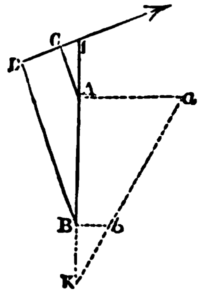
Fig. 60.

A K (or A I = B K); I will be the *required pitch-point* or trace of the line of contact.

The line of connection may be any straight line which traverses I, or whose projection traverses I; as C D. Let A C and B D be perpendicular to the line of connection; then the velocities of the points C and D are identical; and the perpendiculars A C and B D are inversely as the angular velocities of the pieces.

101. **Wheel and Rack.**—Given, at a given instant, the angular velocity of a wheel and the linear velocity of a rack, to find their pitch-point. In fig. 61, let the plane of projection be perpendicular to the axis of the wheel, and let A be the trace of that axis. Draw A I perpendicular to the direction of motion of the rack, and make its length such that a point in the wheel at I shall revolve with a velocity equal to that of the rack; that is to say, make $\mathrm{A\,I} = \dfrac{\text{linear velocity of rack}}{\text{angular velocity of wheel}}$; then I will be *the required pitch-point.*

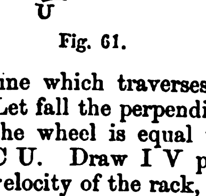
Fig. 61.

The line of connection may be any straight line which traverses I, or whose projection traverses I; as C U. Let fall the perpendicular A C; then the velocity of the point C in the wheel is equal to the component velocity of the rack along C U. Draw I V perpendicular to A I, to represent the whole velocity of the rack, and from V draw V U perpendicular to C U; it is evident that I U is the component velocity along the line of connection; and that A I : A C : : I V : I U.

102. ***Circular Wheels in General.***—In order that, in an *elementary combination of* wheels, or of a wheel and rack, the velocity-

[redacted]

motion; the resultant of these two will be the required other component motion. For example, in fig. 15, let A D be the given resultant motion, and A B the given component; draw D C equal and parallel to A B, and pointing the opposite way; join A C; this will be the required other component: or otherwise, join B D and draw A C equal and parallel to it.

VI. (Fig. 17.) *Given, the vertical projection,* A B, *and the horizontal projection,* A′ B′, *of a straight line representing a motion, to resolve that motion into three rectangular components parallel and perpendicular to the planes of projection.* Let O X be the axis of projection (Article 9, page 4). Draw the straight lines A A′, B B′,

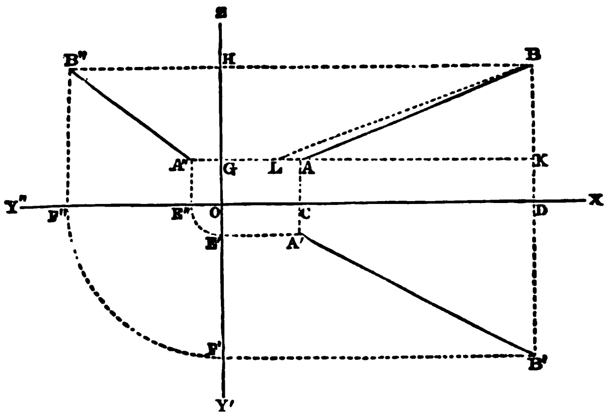

Fig. 17.

cutting the axis of projection (of course at right angles) in C and D. Then through any convenient point, O, in the axis of projection, draw the straight line Z O Y′ at right angles to that axis; and take O Y′ to represent a transverse horizontal axis, and O Z to represent a vertical axis. (The point O is called the *origin.*) Then parallel to X O draw A′ E′ and B′ F′ to meet O Y′, and A G and B H to meet O Z. The three components required will be represented by C D, E′ F′, and G H.

VII. *Given* (in fig. 17), *the vertical projection,* A B, *and the horizontal projection,* A′ B′, *of a straight line representing a motion, to draw a third projection of the same straight line on a vertical transverse plane of projection perpendicular to the first two planes of projection.* Construct fig. 17 as described in the preceding Rule. O Z and O Y′ will be the traces of the third plane of projection. Produce X O towards Y″; then O Y″ will represent the rabatment

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of the axis on a plane perpendicular to it. Draw A I perpendicular to the direction in which the rack is to move, and of a length equal to the given distance; then, about A, with the radius A I, draw a circle, and through I draw a straight line, M N, touching that circle; these will be the required pitch-lines.

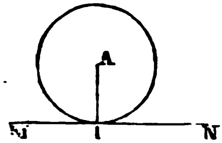

Fig. 64.

105. **Circular Bevel Wheels.**—Given, a pair of axes which intersect each other in a point, and the constant velocity-ratio of two wheels which are to turn about those axes, to draw projections of the pitch-surfaces of those wheels. Let the plane of fig. 65 represent the plane of the two axes; let O A and O B be their positions, and O their point of intersection. Lay off, on any convenient scale, along those axes, the distances O *a* and O *b* respectively proportional to the intended angular velocities (which, in the example shown, are contrary).

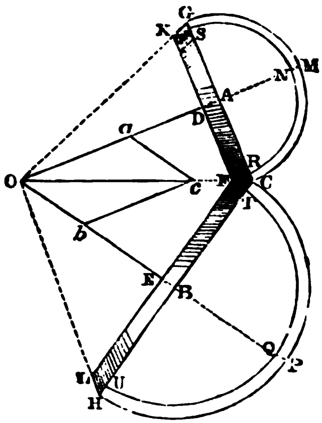

Fig. 65.

Draw *a c* parallel to O *b*, and *b c* parallel to O *a*, cutting each other in *c*; draw the diagonal O *c* C; this will be the line of contact; and the required pitch-surfaces will be parts of two cones described by making O C sweep round O A and O B respectively, and having their common summit at O. O C will be one of the traces of both these cones; and their other traces will be O G, making the angle A O G = A O C; and O H, making the angle B O H = B O C.

In any convenient position on the line of contact, mark a convenient breadth, C F, for the rims of both wheels, so that C F shall be their actual line of contact. Draw C A G and F D K perpendicular to O A, and C B H and F E L perpendicular to O B; then C G K F and C H L F will be the projections of the two wheels on the plane of their axes.

To draw the projection of one-half of each of those wheels on a plane perpendicular to its axis, about A, with the radius A C, draw the semicircle C M G, and with the radius A R = D F draw the semicircle R N S; these will be parts of the pitch-circles of which C A G and F D K are projections, and will form the required projection of one-half of the rim of the wheel whose axis is O A; then, about B, with the radius B C, draw the semicircle

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and in a given interval of time let A B, in fig. 15, page 19, represent the motion of Q relatively to P, and A D the motion of R relatively to P; then A C, found by Rule V. of Article 41, will represent the motion of R relatively to Q.

In all cases whatsoever of relative motion of two bodies, the motion of one relatively to the other is exactly equal and contrary to that of the second relatively to the first. For example, let P and Q be two points; and when P is treated as fixed, let Q move through a given distance in a given direction relatively to P; then if Q is treated as fixed, P moves through the same distance in the contrary direction relatively to Q.

43. **Comparative Motion** (*A. M.*, 358,) is the relation borne to each other by the simultaneous motions of two points, either in the same body or in different bodies, relatively to one and the same fixed point or body. It consists of two elements: the *velocity-ratio*, which is the proportion borne to each other by the distances moved through by the two points in the same interval of time; and the *directional relation*, which is the relation between the directions in which the two points are moving at the same instant.

In the case of two points in a primary piece whose motion is one of translation, the velocity-ratio is that of *equality*, and the directional relation that of *identity;* for all points in such a piece are moving with equal speed in parallel directions at the same instant.

When two points in two different pieces are compared, the comparison may give a different result. For example, let P, as before, stand for the frame of a machine, and Q and R for two moving pieces; and while Q performs relatively to P the motion represented by A B (fig. 15, page 19), let R perform relatively to P the motion represented by A D. Then the *comparative motion* of R and Q consists of the following elements:—

$$\text{the velocity-ratio, } \frac{\text{A D}}{\text{A B}};$$

and the directional relation, represented by the angle B A D.

In most of the cases which occur in mechanism the motion of each point is limited to two directions—forward or backward—in a fixed path; so that the directional relation of two points may often be sufficiently expressed by prefixing the sign + or − to their velocity-ratio, according as their motions are similar or contrary; that is, the sign + denotes that those motions are both forward or both backward; and the sign − that one is forward and the other backward.

We may compare together the different components of the motion of one point, and the resultant motion. For example, in

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the middle of the breadth of the rim of the intended wheel, and let that projection cut O C′ in C′.

I. *To find the radius of the middle pitch-circle, and to draw its projections.* Through B draw B C parallel to O A; through C′ draw C′ C parallel to the axis, cutting B C in C. Join O C; this will be the required radius, and the circle D C G will be the projection of the pitch-circle on the second plane; in G′ C′ produced take G′ D′ = O D = O G = O C; G′ D′ will be the projection of the pitch-circle on the first plane.

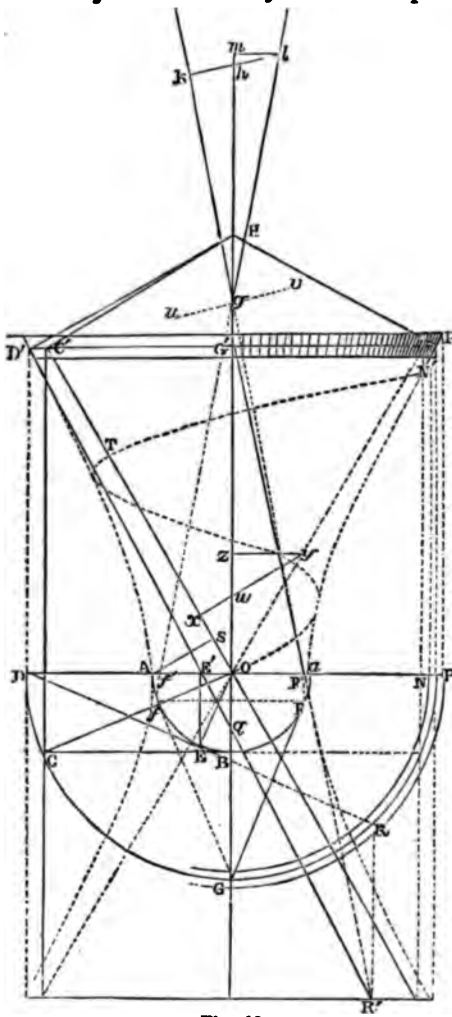

Fig. 68.

D′ is a point in the hyperbolic trace of the hyperboloïd on the first plane; and by the same process any number of points in that trace may be found.

II. *To draw a normal to the pitch-surface in the first plane of projection.* Perpendicular to O C′ draw C′ H, cutting the axis of the wheel in H. This line and O C will be the projections of a normal to the pitch-surface at the point whose projections are C′ and C. Join H D′; this line and O D will be the projections of a normal to the pitch-surface at the point whose projections are D′ and D.

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it is to be held to mean the *place* where the action that drives or that resists a machine is exerted, of what magnitude soever that place may be, whether a surface or a volume. Thus, the driving point in a steam engine comprehends the whole surface of the piston that is pressed upon by the steam which drives the engine; and the working point, where friction is overcome, comprehends the whole of the rubbing surface, and where a heavy body is lifted, the whole volume of that body. Nevertheless, for the sake of convenience in mathematical investigation, such places of the action of driving or resisting forces are often treated on the supposition that they may be represented by single points; for when such points are properly chosen, no error is incurred by making that supposition.

SECTION III.—*Rotation of Primary Pieces.*

45. **Rotation of a Primary Piece.** (*A. M.*, 370-372.)—*Rotation* or *Turning* is the motion of a rigid body when lines in it change their directions; and it is the only kind of motion involving change of the relative positions of the particles of a body that is possible consistently with rigidity; that is to say, with the maintenance of the distance between every pair of particles in the body unchanged. An *axis* of *rotation* is a line in a rigid body whose direction is unchanged by the rotation; and a *fixed axis of rotation* is a line whose position, as well as its direction, is unchanged by the rotation. Every line in a rotating body which is parallel to the axis has its direction unchanged by the rotation. The rotation of a primary piece in a machine always takes place about an axis that is fixed relatively to the frame of the machine; that axis being the geometrical axis, or centre line, of a bearing surface (such as that of the journals or gudgeons of a shaft), whose form is that either of a circular cylinder or of some other surface of revolution The *plane of rotation* is any plane perpendicular to the axis. Every such plane in a rotating body has its position unchanged by the rotation; and straight lines in such a plane—that is, straight lines perpendicular to the axis of rotation—change their directions more rapidly than any other straight lines in the same body.

46. **Speed of Rotation.** (*A. M.*, 373.)—Although in the case of rotation, as well as in that of translation, the principles of pure mechanism are concerned with comparative velocities only, still it is desirable here to state, that the speed with which a rotating body turns is expressed in two different ways. For most practical purposes it is usually stated in turns and fractions of a turn in some convenient unit of time; such as a second, or (more commonly) a minute. For scientific purposes, and for some practical purposes also, *it is expressed in angular velocity*; which means, the angle *swept through in a second* by a line perpendicular to the axis of

Through g draw $u\,g\,v$ parallel to $k\,h\,l$; this will be the rabatment of a tangent to the normal spiral at the point G′.

To find the radius of curvature of a normal spiral at the throat of the hyperboloïd, in O H take O w = O A; draw $x\,w\,y$ perpendicular to O C′, and $y\,z$ parallel to O A; O z will be the required radius of curvature.

The lower part of the figure shows the projection on a plane through the axis, of a pitch-circle equal to the pitch-circle G′ D′, and at the same distance from the throat along the axis in the opposite direction. D E R and D′ E′ R′ are the two projections of one generating line extending from one of those pitch-circles to the other. G′ F′ R′ is the projection of another such generating line. The drawing of a pair of equal pitch-circles may sometimes be useful in the making of patterns for the wheel and for its teeth.

P, P′ and N, N′ are the projections of points in the two edges of the rim of the wheel. When the exact hyperboloidal pitch-surface is to be used, and not merely a tangent cone, those points are to be found by a process similar to that by which the projections D, D′ are found. When a tangent cone is used as an approximation, they are simply the intersections of two planes perpendicular to the axis, with a tangent in the plane of the axis.

VI. *Radius of curvature of hyperbolic trace.*—In constructing the pitch-surface of a skew-bevel wheel, it is sometimes useful to determine the radius of curvature of the hyperbolic trace of that surface on a plane traversing the axis, at the point where that trace cuts the pitch-circle, in order that a circular arc of that radius may, if required, be used as an approximation to a small arc of the hyperbolic curve.

In fig. 68 A, let O X be the axis of the hyperboloïd, O A the radius of its throat, O D an asymptote (being, as before, the projection of a line of contact that is parallel to the plane of projection), and X Y the trace of the plane of the intended pitch-circle. Part of the following process has already been described, but it will be described again here, to make the explanation complete:—Let D be the point where X Y cuts the asymptote. Lay off X E = O A; join D E; and make X Y = D E; then X Y will be the radius of the pitch-circle, and Y a point in the hyperbola. Perpendicular to O D draw D F, cutting the axis in F; join F Y; this will be a normal to the hyperbola at the point Y. Thus far the process has already been described.

Through A draw A B parallel to the axis, cutting the asymptote in B. From B, perpendicular to O B, draw B C, cutting O A produced in C. Then C will be *the centre of curvature*, and A C *the radius of curvature* of the hyperbola at A; that is, at the throat *of the hyperboloïd.*

In X Y, produced both ways as far as may be required, take

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Y H = A O, Y L = A C, and Y G = Y F. In Y F take Y K = A O: join H F and K G. Through L, parallel to F H,

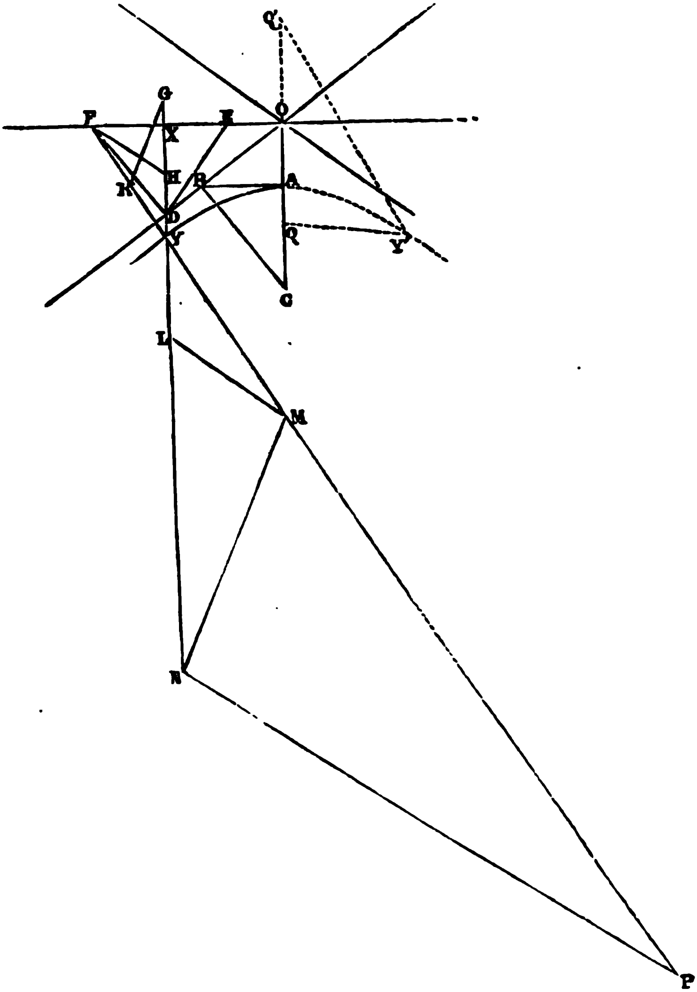

Fig. 68 A.

draw L M, cutting *F Y produced in* M; through M, parallel to *G K, draw M N, cutting X Y L* produced in N; and through N

[redacted]

parallel to F H, draw N P, cutting F Y M produced in P; then P will be *the centre of curvature*, and Y P *the radius of curvature* of the hyperbola at Y.*

VII. *Foci of hyperbolic trace.*—To find, if required, the foci of the hyperbolic trace of the pitch-surface; produce, in fig. 68 A, the straight line O A, in both directions, as far as may be required, and lay off in it O Q = O Q′ = O B. Then Q and Q′ will be the two foci. The well-known property of a hyperbola, by means of which it can be drawn when one point in it and the two foci are given, is, that the difference of the distances from any point in the curve to the foci is a constant quantity; for example, Y′ Q′ — Y′ Q = A Q′ — A Q = 2 A O. Instruments founded on this principle are used for drawing hyperbolas.

107. **Non-Circular Wheels in General.** (*A. M.*, 443.)—Non-circular wheels are used to transmit a variable velocity-ratio between a pair of parallel axes. In fig. 69, let C_1, C_2 represent the axes of such a pair of wheels; T_1, T_2, a pair of points which at a given instant touch each other in the line of contact (which line is parallel to the axes and in the same plane with them); and U_1, U_2, another pair of points which touch each other at another instant of the motion; and let the four points, T_1, T_2, U_1, U_2, be in one plane perpendicular to the two axes and to the line of contact. Then, for every such set of four points, the two following equations must be fulfilled:—

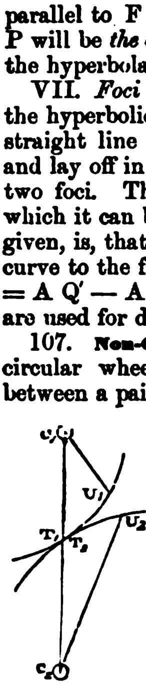

Fig. 69.

$$\left.\begin{array}{c} C_1 U_1 + C_2 U_2 = C_1 T_1 + C_2 T_2 = C_1 C_2; \\ \text{arc } T_1 U_1 = \text{arc } T_2 U_2; \end{array}\right\} \quad (1.)$$

and those equations show the geometrical relations which must exist between a pair of rotating surfaces in order that they may move in rolling contact round fixed axes.

If one of the wheels be fixed and the other be rolled upon it, a point in the axis of the rolling wheel describes a circle of the radius $C_1 C_2$ round the axis of the fixed wheel.

The equations are made applicable to *inside gearing*, by putting − instead of + and + instead of −.

* The algebraical expressions of these operations are as follows:—

Let O A = b; A B = a; O X = x; X Y = y;

X F = m; Y F = n; Y P = ρ; A C = ρ_0; then

$$\rho_0 = \frac{a^2}{b}; \; y = \frac{b}{a}\sqrt{\left(a^2 + x^2\right)};$$

$$m = \frac{b^2 x}{a^2}; \; n = \sqrt{\left(y^2 + m^2\right)}; \; \rho = \rho_0 \frac{n^3}{b^3}.$$

[redacted]

questions in mechanism, there is frequent occasion to measure the lengths of circular arcs, and to lay off circular arcs of given lengths. These processes may be performed by the help of calculation, and of the well-known approximate values of the ratio which the radius and the circumference of a circle bear to each other, viz.:—

$$\frac{\text{circumference}}{\text{radius}} = \frac{710}{113} \text{ nearly} = 6 \cdot 283185 \text{ nearly};$$

$$\frac{\text{radius}}{\text{circumference}} = \frac{113}{710} \text{ nearly} = 0 \cdot 159155 \text{ nearly};$$

but it is often much more convenient in practice to proceed by drawing; and then the following rules are the most accurate yet known:—*

I. (Fig. 22.) *To draw a straight line approximately equal to a given circular arc*, A B. Draw the straight chord B A; produce A to C, making A C = $\frac{1}{2}$ B A; about C, with the radius C B = $\frac{3}{2}$ B A, draw a circle; then draw the straight line A D, touching the given arc in A, and meeting the last-mentioned circle in D; A D will be the straight line required.

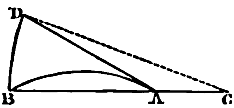

Fig. 22.

The *error* of this rule consists in the straight line being a little shorter than the arc: in fractions of the length of the arc, it is about $\frac{1}{1080}$ for an arc equal in length to its own radius; and it varies as the fourth power of the angle subtended by the arc; so that it may be diminished to any required extent by subdividing the arc to be measured by means of bisections. For example, in drawing a straight line approximately equal to an arc subtending 60°, the error is about $\frac{1}{900}$ of the length of the arc; divide the arc into two arcs, each subtending 30°; draw a straight line approximately equal to one of these, and double it; the error will be reduced to *one-sixteenth* of its former amount; that is, to about $\frac{1}{14400}$ of the length of the arc. The greatest angular extent of the arcs to which the rule is applied should be limited in each case according to the degree of precision required in the drawing.

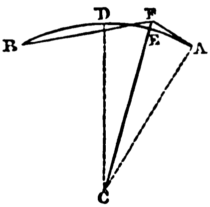

Fig. 23.

II. (Fig. 23.) *To draw a straight line approximately equal to a given circular arc*, A B. (Another Method.) Let C be the centre of the arc. Bisect the arc A B in D, and the arc A D in E; draw the straight

* These rules are extracted from Papers read to the British Association *in 1867, and published in the Philosophical Magazine for September and October of that year.*

[redacted]

with a radius N L = $\frac{3}{4}$ B L, draw a circular arc, cutting the first pitch-line in K. Then B K will be approximately equal in length to B L. Join and measure A K, and in the line of centres take A M = A K. About H, with the radius H M, draw a circular arc, M P, cutting the arc K L P in P; P will be approximately the required point in the second pitch-line.

By repeating the same process, any number of points in the required pitch-line of the second wheel may be found approximately. The error of the two preceding rules, in what may be considered an extreme case—viz., where the pitch-line of the first wheel coincides with the straight tangent B D, and the angles B H F and B A C are each half a right angle (as in designing a roller to roll with a square roller)—is about 0·003 of the length B D to be laid off, and is in excess; the arc B F being too long by that fraction of its length; and the error, in fractions of the arc, varies nearly as the fourth power of the angle subtended by the arc. To ascertain whether the error is sensible, and to correct it by a second approximation, proceed as follows:—

III. *To obtain a closer approximation to the required axis and pitch-line.* Having drawn the pitch-line B F by Rule II., measure its length in subdivisions by Rule I. of Article 51, page 28, and compare that length with B D, so as to ascertain the error. Divide that error by B D, so as to express it in fractions of the required length. Multiply the half-sum of the greatest and least radii by the fraction expressing the ratio of the error to the required length; the product will be a *correction*, which is to be applied to the lengths of the line of centres, A H, and of each of the radii H B, H F, H P, &c., of the second pitch-line; that is to say, if the pitch-line, as at first drawn, is too long, each of those straight lines is to be shortened by having the correction subtracted from it.

For example, in the extreme case already cited, where the first pitch-line is a straight line, B D, perpendicular to A B, and subtending half a right angle at A, and the second pitch-line is to subtend also half a right angle at its axis H, let A B be taken as unity; then we have (to three places of decimals)

$$\text{B C} = \text{B D} = \text{A B} = 1{\cdot}000;$$

A C (coinciding with a straight line from A to D) = 1·414; and the application of Rule I. of this Article gives the following results as first approximations:—

$$\text{A H} = 2{\cdot}267;\ \text{H B} = 1{\cdot}267;\ \text{H F} = 0{\cdot}853.$$

Upon drawing the second pitch-line, B F, by Rule II. of this *Article, and measuring* it in subdivisions, it is found to be too

long by 0·003 of its own length; which being multiplied by $\frac{\text{H B} + \text{H F}}{2} = \frac{2{\cdot}120}{2} = 1{\cdot}060$, gives 0·003 for the correction to be subtracted from the line of centres and from each of the radii of the second pitch-line. Thus are obtained the second approximations,

$$\text{A H} = 2{\cdot}264;\ \text{H B} = 1{\cdot}264;\ \text{H F} = 0{\cdot}850.$$

As examples of non-circular wheels, the following may be mentioned:—

I. An ellipse rotating about one focus rolls completely round in outside gearing with an equal and similar ellipse also rotating about one focus, the distance between the axes of rotation being equal to the major axis of the ellipses, and the velocity-ratio varying from

$$\frac{1 - \text{eccentricity}}{1 + \text{eccentricity}} \text{ to } \frac{1 + \text{eccentricity}}{1 - \text{eccentricity}} \text{ (see Article 108).}$$

II. Lobed wheels, of forms derived from the ellipse, roll completely round in outside gearing with each other (see Article 109).

III. A hyperbola rotating about its farther focus rolls in inside gearing, through a limited arc, with an equal and similar hyperbola rotating about its nearer focus, the distance between the axes of rotation being equal to the axis of the hyperbolas, and the velocity-ratio varying between

$$\frac{\text{eccentricity} + 1}{\text{eccentricity} - 1} \text{ and unity.}$$

IV. Two logarithmic spiral sectors of equal obliquity rotate in rolling contact with each other; or one logarithmic spiral sector rotates in rolling contact with the oblique plane surface of a sliding piece (see Article 110).

108. **Elliptic Wheels.**—The following rules are applicable to the drawing of the pitch-lines of elliptic wheels, and the determination of their comparative motions:—

I. *Given, the angle by which each wheel is alternately to overtake and to fall behind the other, and the length of the line of centres, to draw the ellipse which is the figure of both pitch-lines.*

From a point, B, draw two straight lines, B F = B F′ = half the line of centres, making with each other the given angle F B F′. Join F F′, bisect it in O, produce it both ways, and make O A = O A′ = half the line of centres. Draw B B′ perpendicular to A A′, making O B′ = O B. Then A A′ is the major axis, B B′ the minor axis, O the centre, and F, F′, the two foci of the required ellipse, which may be drawn by means of a suitable instrument or machine, or by the well-known process of putting an endless thread, of a length = 2 A F′ = 2 F A′, round two pins at the foci, and a

[redacted]

pencil equal in diameter to those pins, and moving the pencil round so as to keep the thread tight. In the workshop ellipses of given dimensions can be drawn with great precision by means of the turning lathe, fitted with apparatus to be afterwards referred to.

The wheels are to be centred, as shown in fig. 72, each upon one of its foci. The *fixed foci*, which are thus placed in the axes of the wheels, are marked A, B, in this figure, and the *revolving foci*, C, D. The ellipses in fig. 72 are similar to that in fig. 71, but drawn on one-half of the scale.

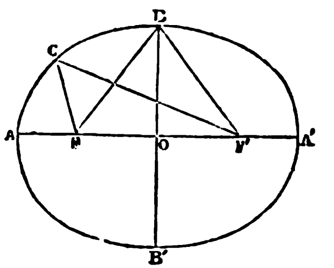

Fig. 71.

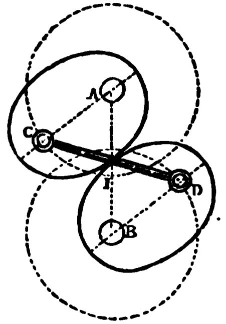

Fig. 72.

II. *To find the angular motions and the angular velocity-ratio corresponding to a given position of the pitch-point.* Suppose both wheels to have started from a position in which A, fig. 71, is the pitch-point, being at the distance A F from the axis of one wheel, and A F' from that of the other, so that the angular velocity-ratio of the second wheel to the first is A F ÷ A F'. Let C be a new position of the pitch-point. Draw C F, C F'. Then the *angular motion* of the first wheel is A F C; that of the second wheel A F' C; the first wheel has *overtaken* the second wheel to the extent represented by the angle F C F' = A F C – A F' C; and the *velocity-ratio* of the second wheel to the first is C F ÷ C F'.

The angular velocity-ratio ranges between the limits $\frac{\text{A F}}{\text{A F}'}$ and $\frac{\text{A F}'}{\text{A F}}$; and its mean value in each half-revolution is *unity;* because each half-revolution is made in the same time by both wheels. The instantaneous velocity-ratio is unity when the pitch-point is at B or B'; because B F = B F'.

III. *Given, at any instant, the position of one of the revolving foci, to find the position of the other revolving focus, and of the pitch-point. In fig. 72, let* A and B be the fixed foci. With a radius equal to *the distance between* the foci, or double eccentricity (F F' in fig.

71), draw circles about A and B. Let C be the given position of one of the revolving foci. Then, with a radius C D = A B (the line of centres), draw a circular arc about C, cutting the circle round B in D; this will be the other revolving focus. Join C D, cutting A B in I; this will be the pitch-point.

If the wheels and their axles overhang the bearings, the revolving foci, being at a constant distance apart, may be connected by means of a link, C D, as shown in fig. 72. This is useful in elliptic toothed wheels of great eccentricity, because of the teeth in certain positions of the wheel being apt to lose hold of each other.

109. **Lobed Wheels** * are wheels such as those shown in figs. 74 and 75, having two, three, or any greater number of equal greatest radii (such as those marked F A″ in fig. 74, and F A‴ in fig. 75), and also of least radii (such as those marked F *a*″ in fig. 74, and F *a*‴ in fig. 75). Fig. 74 represents a two-lobed wheel, and fig. 75 a three-lobed wheel. An elliptic wheel may be regarded as a *one-lobed wheel*. Let the difference between the

Fig. 73.

Fig. 74.

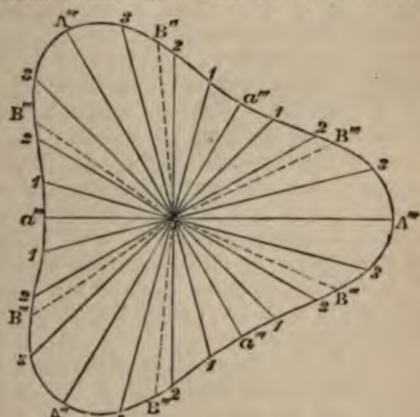

Fig. 75.

greatest and least radii of a lobed wheel be called the *inequality;* so that in an elliptic wheel (fig. 71) the inequality is the distance

* The properties of these *wheels* were discovered by the Reverend W. Holditch.

between the foci, F F′. Then any pair of lobed wheels in which the inequality is the same will, if properly shaped, work together in rolling contact, and that whether their numbers of lobes are many or few, the same or different; and this statement includes one-lobed or elliptic wheels.

The advantage of wheels with two or more lobes is their being self-balanced, which elliptic wheels are not.

The following are the rules for designing lobed wheels:—

I. *Given, in a pair of equal and similar lobed wheels, the angle by which each wheel is alternately to overtake and to fall behind the other wheel, the number of lobes, and the mean radius, to find the inequality, and thence the greatest and least radii.* Multiply the given angle by the number of lobes; then, from a point B‴, fig. 73, draw two lines, B‴ F, B‴ F′, making with each other an angle equal to the product, and make the length of each of them equal to the given mean radius. Draw the straight line F F′; this will be the required *inequality*. Bisect F F′ in O, and produce it both ways; then lay off O A‴ = O a‴ = B‴ F = B‴ F′, the mean radius; then F A‴ = F′ a‴ will be the greatest radius, and F′ A‴ = F a‴ the least radius.

II. *To find any number of points in the pitch-line.* In fig. 73, with the major axis A‴ a‴, and the foci F and F′, draw a semi-ellipse A‴ B‴ a‴. Then, in fig. 75, draw from the centre, F, the straight lines marked F A‴, dividing a complete revolution into as many equal parts as there are to be lobes (in the present case, three). Make each of these lines equal to the greatest radius (F A‴, fig. 73). Bisect the angles between them with the straight lines marked F a‴, fig. 75, and make each of the latter set of lines equal to the least radius (F a‴, fig. 73). Divide the half-revolution in fig. 73 into any convenient number of equal angles by the radiating lines F 1, F 2, &c., meeting the ellipse at 1, 2, &c. Divide each of the angles marked A‴ F a‴ in fig. 75 into the same number of equal parts by radiating lines, and lay off upon them lengths, F 1, F 2, &c., equal to those of the corresponding lines in fig. 73; the points 1, 2, &c., in fig. 75, thus found, will be *points in the required pitch-line.*

III. *To find the positions of the mean radii of the required pitch-line.* Divide the angle A‴ F B‴, in fig. 73 by the number of lobes, and lay off the quotient for each of the angles marked A‴ F B‴ in fig. 75; then make each of the radii F B‴ in fig. 75 equal to F B‴, in fig. 73; these will be the required mean radii.

Remark.—The example in fig. 75 is a three-lobed wheel. The two-lobed wheel of fig. 74 is drawn by a similar process; the ellipse used for finding the radii being A″ B″ a″ in fig. 73; the inequality F F′; and the angle by which each wheel alternately overtakes and

falls behind another equal and similar wheel being one-half of F B'' F', fig. 73.

IV. *To draw the pitch-lines of a set of wheels of different numbers of lobes, all of which shall work with each other in rolling contact.* The inequality must be the same in each wheel. Let F F', fig. 73, be that inequality; and let O be the centre, A''' a''' the major axis, and O B''' the semi-minor axis of the ellipse which serves for finding the radii of one of the set of wheels, which one wheel is given. Divide O B''' into as many equal parts as there are lobes in the given wheel; say, for example, three. To find the figure of a wheel having any other number of lobes (say two), take the point B'' at that number of divisions from O; join F B'', F' B''; lay off O A'' = O a'' = F B'' = F' B''; draw the ellipse A'' B'' a'' with A'' a'' for its major axis, and F and F' for its foci; this will be the ellipse for determining the lengths of the radii of the new (two-lobed) wheel.

The ellipse A' B' a' with the same foci, F F', whose minor semi-axis, O B', is *one* division of O B''', is itself the pitch-line of the one-lobed wheel, which will work in rolling contact with any wheel of the set.

110. **Logarithmic Spiral Sectors or Rolling Cams.**—A pair of logarithmic spiral sectors may be used as rolling cams, to communicate by rolling contact an angular motion of limited extent, in the course of which it is desired that the velocity-ratio shall range between certain limits. The general nature of the figure and position of such a pair of sectors may be represented by fig. 69, page 92.

The only cases in which the dimensions and figures of such sectors can be determined by plane geometry alone, without the aid of calculation, are two, viz.: when the two sectors are equal and similar, so that the sum of the greatest and least radii of each of the two sectors is equal to the line of centres; and when the combination consists of one sector, working with a sliding bar or smooth rack. The following are the rules applicable to such cases:—

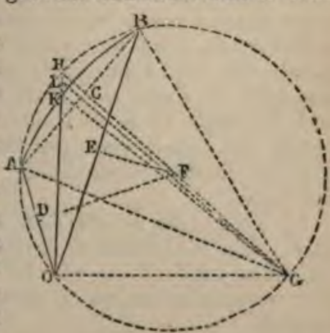

Fig. 76.

I. *Given, in fig.* 76, *the least and greatest radii,* O A *and* O B, *of a logarithmic spiral sector, and the angle* A O B *between them, to find intermediate points in the pitch-line of such a sector, and to draw that pitch-line approximately by means of one or more circular arcs.*

[redacted]

Describe a circle about the triangle O A B; that is to say, bisect any two of the sides of that triangle (C, E, and D being the three points of bisection), and from the points of bisection draw perpendiculars to the sides, meeting in F, which will be the centre of the circle through O, A, and B. Draw the diameter G F C H, bisecting the arc A H B in H and the arc A O G B in G. Join O H (which will be perpendicular to O G, and will bisect the angle A O B); and about G, with the radius G A = G B, draw the circular arc A K B, cutting O H in K. Then K will be a point in the required spiral; and A K B will be the nearest approximation to the spiral arc traversing the three points, A, K and B, that it is possible to make by means of one circular arc only.

To find two additional points, and a closer approximation to the curve, treat each of the triangles O A K and O K B as the triangle O A B was treated; the result will be the finding of two more points in the spiral, situated respectively in the radii which bisect the angles A O K and K O B; and the drawing of two circular arcs, one extending from A to K, and the other from K to B, which will make a closer approximation to the spiral arc than a single circular arc does.

The next repetition of the process will give four additional points and four circular arcs; the next, eight additional points and eight circular arcs; and so on to any degree of precision that may be required.

The radius O K is *a mean proportional* between O A and O B; and this property enables its length to be found by calculation, if required.

The *obliquity* of a logarithmic spiral, being the angle which a tangent to the spiral at a given point makes with a tangent to a circle described about the axis through that point, or the equal angle which a normal to the spiral at the same point makes with a radius drawn from that point to the axis, is uniform in a given spiral. In fig. 76 the equal angles, O A G, O H G, and O B G, are each of them less than the true obliquity of the spiral, and the angle O K G is greater than the true obliquity. To obtain the closest approximation to the true obliquity possible without further subdividing the angle A O B, proceed as follows:—

II. *To find the approximate obliquity.* In H K take H L = $\frac{1}{3}$ H K; join L G; then O L G will be the obliquity, very nearly. In other words, L G will be very nearly parallel to a normal, and perpendicular to a tangent, to the true spiral at the point K.

II A. To find the approximate obliquity (Another method). By *Rule I.* or *Rule II.* of Article 51, page 28, measure the approxi-

mate length of the arc A B in fig. 76. Then, in fig. 77, draw the straight line M N = O B − O A; draw M P perpendicular to M N; and about N, with a radius equal to the approximate length of the arc A B, draw a circular arc, cutting M P in P; join N P; then the angle M P N will be approximately the required obliquity.

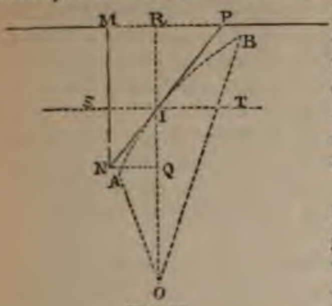

Fig. 77.

III. *Given* (in fig. 76), *one radius*, O K, *in a logarithmic spiral of a given obliquity, to draw approximately a short arc of that spiral through* K. Draw O G perpendicular to O K; draw K G, making O K G = the angle of obliquity, and cutting O K G in G; then, with the radius G K, draw a short circular arc through K.

IV. *To draw the pitch-line of a sliding bar which shall work in rolling contact with a given logarithmic spiral sector*, A O B (fig. 77). From the trace of the axis O draw O Q R perpendicular to the direction in which the bar is to slide, making O Q = O A, and O R = O B. Find the obliquity of the sector by means of one or other of the preceding rules. Let I be any given position of the pitch-point, and let T I S, traversing I perpendicularly to O Q R, be parallel to the direction in which the bar is to slide. Draw the straight line N I P, making the angle S I N = T I P = the obliquity; then draw Q N and R P parallel to T I S, and cutting N I P in N and P respectively. The straight line N I P will be the required pitch-line; and N and P will be the points in it corresponding to A and B respectively in the pitch-line of the sector.

At the instant when the pitch-point is at I, the velocity of the sliding bar is equal to that of the point I in the sector; that is to say, angular velocity × O I; agreeably to the general principle of Article 101, page 84.

The following rules relate to the solution of questions respecting logarithmic spiral sectors by calculation.

V. *Given, two radii of a logarithmic spiral sector* (as O A and O B, fig. 76), *and the angle between them* (A O B), *to find the obliquity of the spiral.* Take the hyperbolic logarithm * of the ratio $\frac{OB}{OA}$; divide it by the angle A O B in

* *Hyperbolic logarithm of a ratio* = common logarithm × 2·3026 *nearly.*

circular measure;* the quotient will be the tangent of the obliquity.

VI. *Given, the least and greatest radii of a logarithmic spiral sector, and the angle between them, to find the lengths of a series of intermediate radii, which shall divide that angle into a given number of equal smaller angles.* Take the difference between the logarithms of the greatest and least radii; divide it by the given number; then, commencing with the logarithm of the least radius, compute by successive additions of the quotient a series of logarithms, increasing by uniform differences up to the logarithm of the greatest radius; these will be the logarithms of the required intermediate radii.

VII. *Given, one radius and the obliquity of a logarithmic spiral, to find the length of a radius making a given angle with the given radius.* Multiply the given angle in circular measure (see first footnote below) by the tangent of the obliquity; to the product add the hyperbolic logarithm of the given radius; the sum will be the hyperbolic logarithm of the required radius;—or otherwise, multiply the product by 0·4343, and to the new product add the common logarithm of the given radius; the sum will be the common logarithm of the required radius.

VIII. *Given, the difference between the greatest and least radii of a logarithmic spiral sector, and the obliquity of its pitch-line, to find the length of its pitch-line.* Multiply the difference of the radii by the cosecant of the obliquity.†

111. **Frictional Gearing.**—To increase that friction or adhesion between a pair of wheels which is the means of transmitting force and motion from one to the other, their surfaces of contact are sometimes formed into alternate ridges and grooves parallel to the

* Reduction of angles to circular measure—

1 degree	= 0·0174533	radius	length,	nearly.
30 degrees	= 0·5236	,,	,,	,,
60 degrees	= 1·0472	,,	,,	,,
90 degrees	= 1·5708	,,	,,	,,

† In symbols, the equations of a logarithmic spiral are as follows:—Let a be the radius from whose directions angles are reckoned; r, any other radius; θ, the angle which r makes with a, in circular measure; ϕ, the obliquity of the spiral; s, the length of the arc from a to r; ρ, the radius of curvature at the end of the radius r. Then

$$r = a e^{\theta \tan \phi}; \ \tan \phi = \frac{1}{\theta} \text{hyp. log.} \frac{r}{a}$$

$$\theta = \text{cotan } \phi \cdot \text{hyp. log.} \frac{r}{a};$$

$$s = (r - a) \text{ cosec } \phi = a \text{ cosec } \phi \left(e^{\theta \tan \phi} - 1\right);$$

$$\rho = r \tan \phi.$$

plane of rotation, constituting what is called *frictional gearing.* Fig. 78 is a cross-section of the rim of a wheel, illustrating the kind of frictional gearing invented by Mr. Robertson. The comparative motion of a pair of wheels thus ridged and grooved is nearly the same with that of a pair of smooth wheels in rolling contact, having cylindrical or conical pitch-surfaces lying midway between the tops of the ridges and bottoms of the grooves.

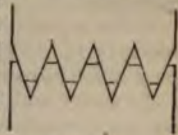

Fig. 78.

The relative motion of the surfaces of contact of the ridges and grooves is a rotatory sliding or grinding motion about the line of contact of the ideal pitch-surfaces as an instantaneous axis; and the angular velocity of that relative grinding motion is equal to the angular velocity of one wheel considered as rolling upon the other as a fixed wheel; which may be found by the principles of Article 77, page 56, and Article 82, page 68.

The angle between the sides of each groove is about 40°; and it is stated that the mutual friction of the wheels is about once and a-half the force with which their axes are pressed towards each other.

SECTION III.—*Of the Pitch and Number of the Teeth of Wheels.*

112. **Relation between Teeth and Pitch-Surfaces—Nature of the Subject.** (*A. M.*, 446.)—The most usual method of communicating motion between a pair of wheels, or a wheel and a rack, and the only method which, by preventing the possibility of the rotation of one wheel unless accompanied by the other, insures the preservation of a given velocity-ratio exactly, is by means of a series of alternate ridges and hollows parallel, or nearly parallel, to the successive lines of contact of the ideal toothless wheels or *pitch-surfaces,* whose velocity-ratio would be the same with that of the toothed wheels. The ridges are called *teeth;* the hollows, *spaces.* The teeth of the driver push those of the follower before them, and in so doing sliding takes place between them in a direction across their lines of contact.

The properties of pitch-surfaces and pitch-lines, and the art of designing them, have been explained in the preceding section. The figures of teeth depend on the principles of sliding contact, which belong to the ensuing section. The present section relates to questions connected with the manner in which the pitch-line of a wheel is divided by the acting surfaces of its teeth, without reference to the figures of those surfaces; for such questions do not require the *principles of sliding* contact for their solution.

113. ***Pitch Defined.*** (*A. M.*, 447.)—The distance, measured

[redacted]

along the pitch-line, from the front of one tooth to the front of the next, is called the PITCH.

114. **General Principles.** (*A. M.*, 447.)—The pitch, and the number of teeth in wheels, are regulated by the following principles:—

I. In wheels which rotate continuously for one revolution or more, it is obviously necessary *that the pitch should be an aliquot part of the circumference of the pitch-line.*

In racks, and in wheels which reciprocate without performing a complete revolution, this condition is not necessary. Such wheels are called *sectors*, as has been already stated.

II. In order that a pair of wheels, or a wheel and a rack, may work correctly together, it is in all cases essential *that the pitch should be the same in each.*

III. Hence, in any pair of wheels which work together, *the numbers of teeth in a complete circumference are inversely as the numbers of whole revolutions in a given time;* or, in other words, *directly as the times of revolution.*

IV. Hence, also, in any pair of wheels which rotate continuously for one revolution or more, the ratio of the times of revolution (being equal to that of the numbers of teeth), and its reciprocal, the ratio of the numbers of revolutions in a given time, *must both be expressible in whole numbers.*

V. In circular wheels, everything stated in the preceding principles regarding the ratio of the numbers of revolutions in a given time (in other words, of the *mean angular velocity-ratio*) is true also of the angular velocity-ratio at every instant.

115. **Frequency of Contact—Hunting-Cog.**—Let n, N, be the respective numbers of teeth in a pair of wheels, N being the greater. Let t, T, be a pair of teeth in the smaller and larger wheel respectively, which at a particular instant work together. It is required to find, first, how many pairs of teeth must pass the pitch-point before t and T work together again (let this number be called a); secondly, with how many different teeth of the larger wheel the tooth t will work at different times (let this number be called b); and thirdly, with how many different teeth of the smaller wheel the tooth T will work at different times (let this be called c).

CASE 1. If n is a divisor of N,

$$a = \mathrm{N};\ b = \frac{\mathrm{N}}{n};\ c = 1 \ldots\ldots\ldots\ldots\ldots\ldots (1.)$$

CASE 2. If the greatest common divisor of N and n be d, a number less than n, so that $n = m\,d$, $\mathrm{N} = \mathrm{M}\,d$, then

$$a = m\,\mathrm{N} = \mathrm{M}\,n = \mathrm{M}\,m\,d;\ b = \mathrm{M};\ c = m \ldots\ldots (2.)$$

CASE 3. If N and n be prime to each other,

$$a = \mathrm{N}\,n;\ b = \mathrm{N};\ c = n \ldots\ldots\ldots\ldots\ldots\ldots (3.)$$

It is considered desirable by millwrights, with a view to the preservation of the uniformity of shape of the teeth of a pair of wheels, that each given tooth in one wheel should work with as many different teeth in the other wheel as possible. They therefore study to make the numbers of teeth in each pair of wheels which work together such as to be either prime to each other, or to have their greatest common divisor as small as is possible consistently with the purposes of the machine.

When the ratio of the angular velocities of two wheels, being reduced to its least terms, is expressed by numbers less than those which can be given to wheels in practice, and it becomes necessary to employ multiples of those numbers by a common multiplier, which becomes a common divisor of the numbers of teeth in the wheels, millwrights and engine-makers avoid the evil of frequent contact between the same pairs of teeth, by giving one additional tooth, called a *hunting-cog*, to the larger of the two wheels. This expedient causes the velocity-ratio to be not exactly but only approximately equal to that which was at first contemplated; and therefore it cannot be used where the exactness of certain velocity-ratios amongst the wheels is of importance, as in clockwork.

116. **Smallest Pinion.**—The *smallest* number of teeth which it is practicable to give to a pinion (that is, a small wheel), is regulated by principles which will appear when the forms of teeth are considered. The following are the least numbers of teeth which can be *usually* employed in pinions having teeth of the three classes of figures named below, whose properties will be explained in the sequel:—

I.	Involute teeth,............................	25
II.	Epicycloidal teeth,.......................	12
III.	Round teeth, or *staves*,..................	6

117. **Arithmetical Rules.**—For convenience sake the following arithmetical rules are here given, as being useful in the designing of toothed gearing.

I. *To find the prime factors of a given number.* Try the prime numbers, 2, 3, 5, 7, 11, &c., as divisors in succession, until a prime number has been found to divide the given number without a remainder; then try whether and how many times over the quotient is again divisible by the same prime number, so as to obtain a quotient not divisible again by the same prime number; then try the division of that quotient by the next greater prime number; and so on until a quotient is obtained which is itself a prime number; that is, a number not divisible by any other number except 1. This final *quotient and the series of divisors will be the prime factors of the given number.* To test the accuracy of the process, multiply

[redacted]

all the prime factors together; the product should be the given number.

II. *To find the greatest common measure* (otherwise called the *greatest common divisor*) *of two numbers.* Divide the greater number by the less, so as to obtain a quotient, and a remainder less than the divisor; divide the divisor by the remainder as a new divisor; that new divisor by the new remainder; and so on, until a remainder is obtained which divides the previous divisor without a remainder. That last remainder will be the required greatest common measure.

If the last remainder is 1, the two numbers are said to be "prime to each other."

Example.—Required, the greatest common measure of 1420 and 1808.

Divisor, 1420) 1808 (1, Quotient.
1420
Remainder, 388) 1420 (3, Quotient.
1164
Remainder, 256) 388 (1, Quotient.
256
Remainder, 132) 256 (1, Quotient.
132
Remainder, 124) 132 (1, Quotient.
124
Remainder, 8) 124 (15, Quotient.
120
Remainder, 4) 8 (2, Quotient.

The last remainder, 4, is the required greatest common measure.

III. To reduce the ratio of two numbers to its least terms, divide both numbers by their greatest common measure.

$$\text{For example, } \frac{1808 \div 4}{1420 \div 4} = \frac{452}{355}.$$

IV. *To express the ratio of two numbers in the form of a continued fraction.* Let A be the lesser of the two numbers, and B the greater; and let a, b, c, d, &c., be the quotients obtained during the process of finding the greatest common measure of A and B. Then, in the equation

$$\frac{B}{A} = a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d + \&c.,}}}$$

the right-hand side is the continued fraction required.

To save space in printing, a continued fraction is often arranged as follows:—

$$a + \frac{1}{b+}\,\frac{1}{c+}\,\frac{1}{d+}\ \&c.$$

The ratio of two incommensurable quantities is expressed by an endless continued fraction. For example, the ratio of the diagonal to the side of a square is expressed by $1 + \frac{1}{2+}\,\frac{1}{2+}\,\frac{1}{2+}\,\frac{1}{2+}$ &c., without end.

V. *To form a series of approximations to a given ratio.* Express the ratio in the form of a continued fraction. Then write the quotients in their order; and in a line below them write $\frac{0}{1}$ to the left of the first quotient, and $\frac{1}{0}$ directly under the first quotient. Then calculate a series of fractions by the following rule:—Multiply the first quotient by the numerator of the fraction that is below it, and add the numerator of the fraction next to the left; the sum will be the numerator of a new fraction: multiply the first quotient by the denominator of the fraction that is below it, and add the denominator of the fraction that is next to the left; the sum will be the denominator of the new fraction; then write that new fraction under the second quotient, and treat the second quotient, the fraction below it, and the fraction next to the left, as before, to find a fraction which is to be written under the third quotient, and so on. For example:

Quotients,....a, b, c, d, &c.

Fractions, $\frac{0}{1}$, $\frac{1}{0}$, $\frac{n}{m}$, $\frac{n'}{m'}$, $\frac{n''}{m''}$;

$$\frac{n}{m} = \frac{0+a}{1+0} = \frac{a}{1};\ \frac{n'}{m'} = \frac{1+b\,n}{0+b\,m};\ \frac{n''}{m''} = \frac{n+c\,n'}{m+c\,m'};\ \&c.$$

To take a particular case; let the given ratio be as before, $\frac{452}{355}$; then we have the following series:—

on a plane perpendicular to the axis, of the root-circle through F, the pitch-circle through I, and the addendum-circle through E. Draw also about a the pitch-circle at the throat of the hyperboloïd, and let $a\,i''$ be its radius. Through i draw a straight line, $i\,i''\,i'$, so as to touch this *throat pitch-circle,* and let that straight line cut the circle $i\,i'$ in i and i'. Draw the straight lines $f\,f''\,f'$ and $e\,e''\,e'$ parallel to $i\,i''\,i'$. Then these three parallel lines will be the *projections of the three middle lines* before mentioned, *on a plane perpendicular to the axis.*

Describe about a two circles touching $f\,f''\,f'$ and $e\,e''\,e'$ respectively. These will be respectively *the root-circle and the addendum-circle at the throat of the hyperboloïd.* The roots and crests of all the teeth lie in a pair of hyperboloïdal surfaces traversing this pair of circles, and traversing also the pair of circles through F and E.

The projection, on a plane traversing the axis, of the middle line of the tooth on the pitch-surface is the tangent I I″ already found, the points I″ and i'' being in one straight line parallel to a A. To find the corresponding projections of the other two middle lines, there are two methods.

First Method.—From the points of contact f'' and e'', parallel to a A, draw f'' F″ and e'' E″, cutting $a\,i$ in F″ and E″ respectively. Join F F″ and E E″. These will be the required projections.

Second Method.—Lay off on the axis, a C′ $= a$ C, and a A′ $=$ a A, and draw C′ I′ parallel to C I: then C′ I′ will be part of the projection of a pitch-circle equal to C I. From i', parallel to A a A′, draw i' I′, cutting C′ I′ in I′. Then i' and I′ will be the two projections of one pitch-point, and II″ I′ will be one straight line. Join A′ I′. This will be the projection of a normal to the pitch-surface at I′. Through f' and e' (which lie in one radius, $a\,f'\,i'\,e'$) draw f' F′ and e' E′ parallel to A a A′, cutting A′ I′ in F′ and E′ respectively. Join F F′ and E E′. These will be the required projections of the middle lines at the root and crest of the tooth respectively.

IV. *Complete Projection of a Tooth on a Plane Normal to the Axis.*—Let the plane of projection in fig. 105 be normal to the axis of the wheel, and (as in fig. 103) let a be the axis; let the circles $e\,e'$, $i\,i'$, and $f\,f'$, be the projections of the middle addendum-circle, middle pitch-circle, and middle root-circle of the intended wheel; let the circles through e'', i'', and f'' be the corresponding circles at the throat of the pitch-surface; and let the parallel straight lines $e\,e''\,e'$, $i\,i'\,i''$, $f\,f''\,f'$, be the projections of the middle lines of a tooth at the crest, pitch-surface, and root, drawn according to the preceding rules.

At the end of the radius $a\,f\,i\,e$ construct, by the rules already given, *the projection* of the trace of the tooth upon the middle normal

that is to say, the velocity-ratio of the last and first axes is the ratio of the product of the numbers of teeth in the drivers to the product of the numbers of teeth in the followers; and it is obvious, that so long as the same drivers and followers constitute the train, the *order* in which they succeed each other does not affect the resultant velocity-ratio.

Supposing all the wheels to be in outside gearing, then, as each elementary combination reverses the direction of rotation, and as the number of elementary combinations, $m - 1$, is one less than the number of axes, m, it is evident that if m is odd, the direction of rotation is preserved, and if even, reversed.

It is often a question of importance to determine the numbers of teeth in a train of wheels best suited for giving a determinate velocity-ratio to two axes. It was shown by Young, that to do this with the *least total number of teeth*, the velocity-ratio of each elementary combination should approximate as nearly as possible to 3·59. This would in some cases give too many axes; and as a convenient practical rule it may be laid down, that from 3 to 6 ought to be the range of the velocity-ratio of an elementary combination in wheelwork.*

Let $\frac{B}{C}$ be the velocity-ratio required, reduced to its least terms, and let B be greater than C.

If $\frac{B}{C}$ is not greater than 6, and C lies between the prescribed minimum number of teeth (which may be called t), and its double $2t$, then one pair of wheels will answer the purpose, and B and C will themselves be the numbers required. Should B and C be inconveniently large, they are if possible to be resolved into factors, and those factors, or, if they are too small, multiples of them, used for the numbers of teeth. Should B or C, or both, be at once inconveniently large, and prime, or should they contain inconveniently large prime factors, then, instead of the exact ratio $\frac{B}{C}$,

* The following are some examples of the results of Young's rule, the first line containing velocity-ratios, and the second, the numbers of elementary combinations of wheels suited to give velocity-ratios intermediate between the numbers in the first line:—

1	7	24	88	315	1132	4064	14596
1	2	3	4	5	6	7	

The following are examples of the results of the modified rule, that the lowest of the velocity-ratios for each elementary combination should range from 3 to 6:—

1	6	36	216	1296	7776
1	2	3	4	5	

some ratio approximating to that ratio, and capable of resolution into convenient factors, is to be found by the method of continued fractions (see Article 117, page 106); also Willis *On Mechanism*, pages 223 to 238).

Should $\frac{B}{C}$ be greater than 6, the best number of elementary combinations is found by dividing by 6 again and again till a quotient is obtained less than unity, when the number of divisions will be the required number of combinations, $m - 1$.

Then, if possible, B and C themselves are to be resolved each into $m - 1$ factors, which factors, or multiples of them, shall be not less than t, nor greater than $6t$; or if B and C contain inconveniently large prime factors, an approximate velocity-ratio, found by the method of continued fractions, is to be substituted for $\frac{B}{C}$, as before. When the prime factors of either B or C are fewer in number than $m - 1$, the required number of factors is to be made up by inserting 1 as often as may be necessary. In multiplying factors that are too small to serve for numbers of teeth, prime numbers differing from those already amongst the factors are to be preferred as multipliers; and in general, where two or more factors require to be multiplied, different prime numbers should be used for the different factors.

So far as the resultant velocity-ratio is concerned, the *order* of the drivers N, and of the followers n, is immaterial; but to secure equable wear of the teeth, as explained in Article 115, page 104, the wheels ought to be so arranged that for each elementary combination the greatest common divisor of N and n shall be either 1, or as small as possible; and if the preceding rules have been observed in the choice of multipliers, this will be ensured by so placing each driving wheel that it shall work with a following wheel whose number of teeth does not contain any of the same multipliers; for the original numbers B and C contain no common factor except 1.

The following is an example of a case requiring the use of additional multipliers:—Let the required velocity-ratio, in its least terms, be

$$\frac{B}{C} = \frac{360}{7}.$$

To get a quotient less than 1, this ratio must be divided by 6 three times, therefore $m - 1 = 3$. The prime factors of 360 are $2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$; these may be combined so as to make three factors in various different ways; and the preference is to be given

to that which makes these factors least unequal, viz., $5 \cdot 8 \cdot 9$. Hence, resolving numerator and denominator into three factors each, we have

$$\frac{B}{C} = \frac{5 \cdot 8 \cdot 9}{1 \cdot 1 \cdot 7}.$$

It is next necessary to multiply the factors of the numerator and denominator by a set of three multipliers. Suppose that the wheels to be used are of such a class that the smallest pinion has 12 teeth, then those multipliers must be such that none of their products by the existing factors shall be less than 12; and for reasons already given, it is advisable that they should be different prime numbers. Take the prime numbers, 2, 13, 17 (2 being taken to multiply 7); then the numbers of teeth in the followers will be

$$13 \times 1 = 13;\ 17 \times 1 = 17;\ 2 \times 7 = 14.$$

In distributing the multipliers amongst the factors of the numerator, let the smallest multiplier be combined with the largest factor, and so on; then we have

$$17 \times 5 = 85;\ 13 \times 8 = 104;\ 2 \times 9 = 18.$$

Finally, in combining the drivers with the followers, those numbers are to be combined which have no common factor; the result being the following train of wheels:—

$$\frac{85}{14} \cdot \frac{18}{13} \cdot \frac{104}{17} = \frac{360}{7}.$$

119. **Diametral and Radial Pitch.**—The *diametral pitch* of a circular wheel is a length bearing the same proportion to the pitch proper, or *circular pitch*, that the diameter of a circle bears to its circumference; and the *radial pitch* is half the diametral pitch. In other words, the diametral pitch is to be found by dividing the diameter of the pitch-circle by the number of teeth in the whole circumference, and the radial pitch by dividing the radius by the same number. In symbols, let p be the pitch, properly so called, or circular pitch, as measured on the pitch circle, r the radius of the pitch circle, or *geometrical radius*, and n the number of teeth; q the diametral pitch, and $\frac{q}{2}$ the radial pitch; then

$$q = \frac{113}{355} p = \frac{2\,r}{n};\ 2\,r = n\,q;\ p = \frac{355}{113} q;$$

$$\frac{q}{2} = \frac{113}{710} p = \frac{r}{n};\ r = \frac{n\,q}{2};\ p = \frac{710}{113} \cdot \frac{q}{2}.$$

[redacted]

and the method of drawing those arcs is as follows:—Let fig. 107 represent the trace of part of a wheel with its teeth, that wheel being the smallest wheel of a set that are to be capable of gearing together; because the smallest wheel of such a set requires the greatest addendum: let C be the centre, A A the addendum-circle,

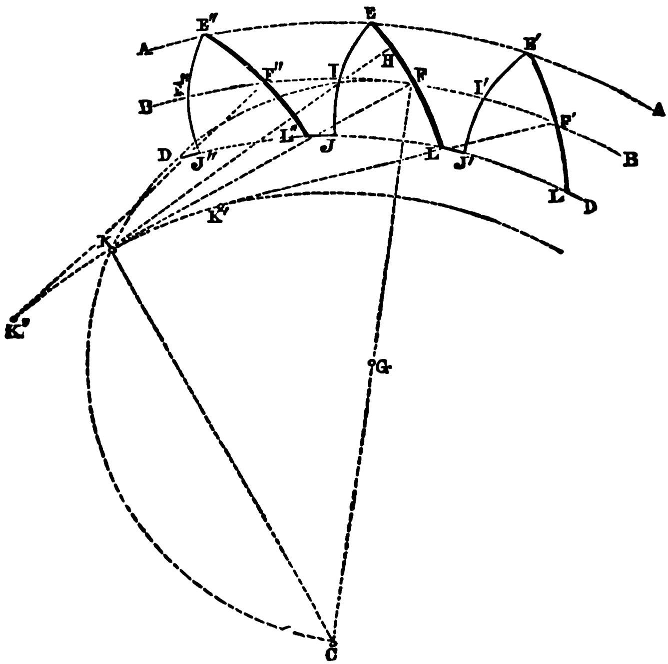

Fig. 107.

B B the pitch-circle, D D the root-circle, and let E" I" J", E I J, E' I' J', be the fronts of teeth designed according to the proper rules, and F", F, F', the pitch-points of the backs of those teeth. To any one of those *back pitch-points*, as F, draw the radius C F; bisect C F in G, and about G draw the semicircle F K C. Draw a straight line, H K, perpendicular to and bisecting the distance, E F, between the crest E and back pitch-point F; and let that straight line cut the semicircle in K. About the centre C, with the radius C K, draw the circle K" K K'; this will be the *base-circle* of the required involutes (see Article 131, page 121).

To draw the circular arcs approximating to those involutes, *lay off, from the* back pitch-points to the base-circle, the equal *distances F" K" =* F' K' = F K, &c; and about the respective

[redacted]

upper part of the figure is a projection of the rim of a wheel with stepped teeth on a plane parallel to the axis, and the lower part is a projection on a plane perpendicular to the axis. A wheel thus formed resembles in shape a series of equal and similar toothed discs placed side by side, with the teeth of each a little behind those of the preceding disc. In such a wheel, let p be the circular pitch, and n the number of steps. Then the path of contact, the addendum, and the extent of sliding, are those due to the *divided* pitch $\frac{p}{n}$, while the strength of the teeth is that due to the thickness corresponding to the *total* pitch p; so that the smooth action of small teeth and the strength of large teeth are combined. The action of small teeth is smoother and steadier than that of large teeth, because they can be made to approximate more closely to the exact theoretical figure; and also because the sliding motion of one tooth upon another is of less extent. In the example shown in fig. 108 there are four steps, so that the divided pitch is one-fourth of the total pitch; and the path of contact (E I F, in the lower part of the figure) is of the length suited to the divided pitch, being only one-fourth of the length which would have been required had the fronts of the teeth not been stepped.

151. **Helical Teeth,** also invented by Dr. Hooke with the same object, are teeth whose fronts, instead of being parallel to the line of contact of the pitch-cylinders of a pair of spur-wheels, cross that line obliquely, so as to be of a screw-like or helical form: in other words, they are teeth of the figure of short portions of *screw-threads* (Article 58, page 36); the trace of each thread on a plane perpendicular to the axis being similar to that of a stepped tooth, as shown in the lower part of fig. 108. Fig. 108 A shows a projection of the rim of a wheel with helical teeth on a plane parallel to the axis.

In order that a pair of wheels with parallel axes and helical teeth may gear correctly together, the teeth, besides being of the same circular pitch, must have the same transverse obliquity; and if in outside gearing, they must be right-handed on one wheel and left-handed on the other. If in inside gearing, they must be either right-handed or left-handed on both wheels. In fig. 108 A the teeth are left-handed. In wheel-work of this kind the contact of each pair of teeth commences at the foremost end of the helical fronts, and terminates at the aftermost end; and the rims of the wheels are to be made of such a breadth that the contact of one pair of teeth shall not terminate until that of the next pair has commenced.

Helical teeth are open to the objection that they exert a laterally *oblique pressure,* which tends to increase friction.

When, in designing a skew-bevel wheel, a portion of the tangent *cylinder at the throat* of the hyperboloïd (Article 106, page 87;

and Article 85, page 73) is used as an approximation to the true pitch-surface, the teeth of that wheel become screw-threads, having a transverse obliquity determined by the principles of Article 147, page 152; and, as has been already stated in the article referred to, they are either right-handed or left-handed in both wheels.

152. **Screw and Nut.**—The figure of a true screw, external or internal, and the motion of a screw working in a corresponding screw-shaped bearing, have been described in Articles 57 to 66, pages 36 to 42. In the elementary combination of an *external and internal screw*, more commonly called a *screw and nut*, the two pieces have threads, one external and the other internal, of similar figures and equal dimensions, so as to fit each other truly; and one of them turns about their common axis without translation, while the other slides parallel to that axis without rotation. The best form of section for the threads is rectangular. The comparative motion is, that the sliding piece advances through a distance equal to the pitch (viz., the "*total axial pitch*") during each revolution of the turning piece. If the threads are $\left\{\begin{array}{c}\text{right-handed,}\\ \text{left-handed,}\end{array}\right\}$ the sliding piece is made to move towards an observer at one end of the axis by $\left\{\begin{array}{c}\text{right-handed}\\ \text{left-handed}\end{array}\right\}$ rotation, and to move from him by $\left\{\begin{array}{c}\text{left-handed}\\ \text{right-handed}\end{array}\right\}$ rotation, of the turning piece. The combination belongs to Mr. Willis's Class A, because the velocity-ratio is constant; and the extent of the motion is limited by the length of the screw.

153. **Screw Wheel-Work in General.**—Screw wheel-work consists of wheels with cylindrical pitch-surfaces, having screw-threads or helical teeth instead of ordinary teeth. One case of screw-gearing has been described in Article 151, page 156—viz., that in which the axes are parallel. The cases to which this and the following articles relate are those in which the axes are not parallel; so that the pitch-surfaces in an elementary combination are a pair cylinders touching each other in *one pitch-point*, like those represented in Article 85, fig. 55, page 73. The pitch-point (O′, fig. 55) is obviously in the common perpendicular of the two axes (F′ G′, fig. 55); and there is one straight line traversing the pitch-point (O′ C′, fig. 55), which is a tangent at once to the two pitch-cylinders and to the acting surfaces or fronts of each pair of threads at the instant when those surfaces touch each other at the pitch-point: that straight line may be called the LINE OF CONTACT. The *angles of inclination* of the screw-threads to the two axes (see Article 63, page 40) are equal respectively to the angles made by the line of contact with those axes. The PITCH-CIRCLES of the two screws are the two circular *sections* of the pitch-cylinders which traverse *the pitch-point. The* PLANE OF CONNECTION, OR PLANE OF

[redacted]

ACTION, is a plane traversing the pitch-point normal to the line of contact: that plane, of course, traverses the common perpendicular of the axes.

When the line of contact is found by the rule given in Article 84, page 71, the cylindrical pitch-surfaces represent the tangent-cylinders at the throats of a pair of hyperboloïds; and the screw-threads are approximations to the skew-bevel teeth suited for that combination, as already stated in Article 151, page 156. But in many cases the line of contact has positions greatly differing from this; and then the comparative motion becomes different from that of a pair of skew-bevel wheels; the object of screw-gearing in such cases being to obtain, with a given pair of cylindrical pitch-surfaces, a velocity-ratio of rotation independent of the radii of those surfaces; and such is the difference between approximate skew-bevel gearing and screw gearing in general.

In every elementary combination in screw wheel-work, each of the two pieces is at once a screw and a wheel; but it is customary, when their diameters are very different, to call that which has the smaller diameter the ENDLESS SCREW, or WORM, and that which has the greater diameter the WORM-WHEEL. For example, in fig. 111 (farther on) a' is the worm, or endless screw, and A' the worm-wheel. The word "endless" is used because of the extent of the motion being unlimited.

Screw wheel-work belongs to Mr. Willis's Class A, the velocity-ratio being constant.

The following are the general principles of elementary combinations in screw wheel-work:—

I. The angular velocities of the two screws are inversely, and their times of revolution directly, as the numbers of threads; whence it follows that the angular velocity-ratio must be expressible in whole numbers, as in the case of ordinary toothed wheels.

II. The *divided normal pitch* (see Article 66, page 42), as measured on the pitch-cylinders, must be the same in two screws that gear together.

III. The *common component* of the velocities of a pair of points in the two screws at the instant when those two points touch each other and pass the pitch-point, is perpendicular to the line of contact and to the common perpendicular of the axes; in other words, it coincides with the intersection of the plane of connection and the common tangent-plane of the two pitch-cylinders.

IV. The *circular* or *circumferential pitches* of the two screws (Article 42, page 66), as measured on their pitch-cylinders, are proportional to the total velocities of points (called the *surface velocities*) in those cylinders; and they bear the same proportion to the divided normal pitch that those total velocities bear to their common component.

V. The *relative transverse sliding* of a pair of threads that are in action takes place along the line of contact.

It will be shown in the next article that for a given pair of axes and a given angular velocity-ratio the relative transverse sliding is least when the pitch-cylinders are the tangent-cylinders at the throats of a pair of skew-bevel hyperboloïds.

154. **Screw Wheel-Work—Rules for Drawing.**—In figs. 109 and 110 the plane of projection is supposed to be the common tangent-plane of the two pitch-cylinders; and I represents the pitch-point; which is also the trace and projection of the common perpendicular of the two axes.

I. *Given, the projections of the two axes, the angular velocity-ratio, and the radii of the two pitch-cylinders, to find the proportionate values of their surface-velocities, and the proportionate value and direction of the velocity of transverse sliding.* The two cylinders may be called respectively A and *a*.

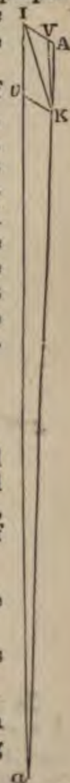

Fig. 109.

In fig. 109, let I A and I *a* represent the projections of the two axes. Along those projections lay off lengths I A, I *a*, proportional to the two angular velocities of rotation, and pointing in the direction in which an observer must look from I in order to make both rotations seem right-handed. Draw the straight line A *a*, and divide it at K into two parts inversely proportional to the radii of the two pitch-cylinders; in other words, let B and *b* denote the radii of the cylinders A and *a* respectively, so that B + *b* is the length of the common perpendicular, or line of centres; and let K divide A *a* in the following proportion:—

$$B + b : B : b$$
$$:: A a : K a : K A$$

Complete the parallelogram I V K *v*; then I V, I *v*, and the diagonal I K, will be respectively proportional and perpendicular to the surface velocity of the cylinder A, the surface velocity of the cylinder *a*, and the velocity of relative transverse sliding at the pitch-point I.

Or otherwise, by calculation; let $\frac{a}{A}$ be the ratio of the angular velocities, and $\frac{b}{B}$ that of the radii; then $\frac{a\,b}{A\,B}$ is obviously the ratio of the surface velocities.

It is obvious that for a given pair of axes and a given pair of angular velocities the velocity of transverse sliding is least when I K is perpendicular to A *a*. But A *a* is parallel to the line of contact of a pair of hyperboloïdal pitch-surfaces for skew-bevel wheels having the given

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velocity-ratio; and this is the demonstration of the statement in the preceding article, that screws which coincide approximately with skew-bevel wheels give the least possible transverse sliding of the threads for a given pair of axes and a given velocity-ratio (see page 159).

The proportionate value of the *common component of the surface velocities* may be represented by the length of a perpendicular let fall from either V or *v* upon I K; but the next rule gives a more convenient way of representing both it and the transverse sliding.

II. *To draw the line of contact, and to find the proportions borne to the surface velocities by their common component, and by the transverse sliding; also the proportions borne to each other by the circular pitches, the divided axial pitches, and the divided normal pitch.*

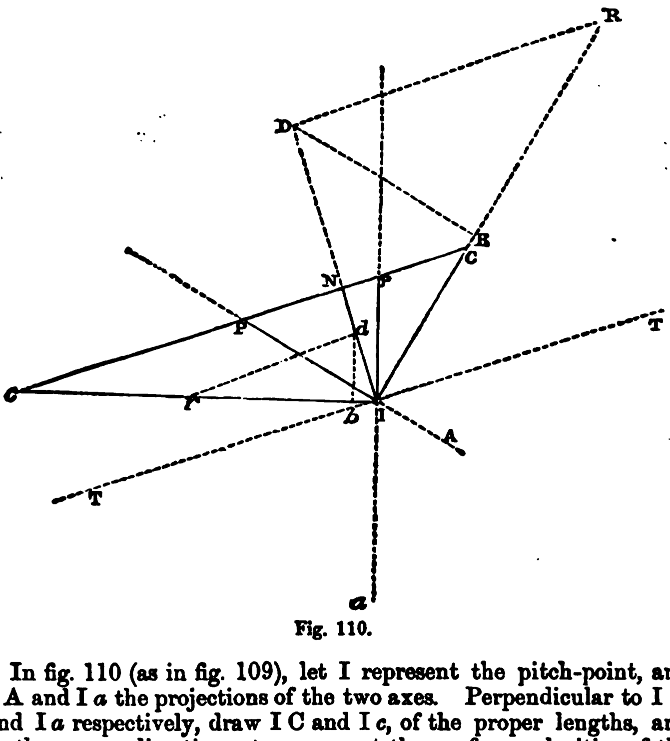

Fig. 110.

In fig. 110 (as in fig. 109), let I represent the pitch-point, and I A and I *a* the projections of the two axes. Perpendicular to I A and I *a* respectively, draw I C and I *c*, of the proper lengths, and in the proper directions, to represent the surface velocities of the two pitch-cylinders at the point I; draw the straight line C *c*, cutting the projections of the two axes in P and *p* respectively;

and upon C c let fall the perpendicular I N (which will obviously be parallel to I K in fig. 109). Through I draw T I T parallel to C c.

Then T I T will be *the line of contact;* I N will represent *the common component of the surface velocities* (and will also be the trace of the plane of connection); C c will represent *the velocity of transverse sliding;* and the proportions of the several divided pitches will be as follows:—

I C : circular pitch of A.
: : I c : circular pitch of a.
: : I P : divided axial pitch of A.
: : I p : divided axial pitch of a.
: : I N : divided normal pitch of both screws.

The figure may be regarded as part of the *development* of both screws upon the common tangent plane of their pitch-cylinders. (See Article 63, page 40. As to Racks, see Addendum, page 289.)

The *absolute lengths* of the circular pitches are found by dividing the pitch-circles into suitable numbers of equal parts, precisely as in the case of spur-wheels (see Articles 112 to 121, pages 103 to 114); and from them, by the aid of the proportions given by fig. 110, the absolute lengths of the divided axial pitches and of the divided normal pitch are easily found. For the total axial pitch of either screw, multiply the divided axial pitch by the number of threads.

III. *To find the radii of curvature of the normal screw-lines.* The normal helix, or normal screw-line (see Article 65, page 41), of each of the two screws touches I N at the pitch-point I; and the plane of connection of which I N is the trace is the common osculating plane of the two normal screw-lines at I. Their radii of curvature at that point both coincide with the common perpendicular of the axes. The rule for finding such radii (Articles 64 and 65, page 41), when applied to this case, takes the following form:—On I C lay off I B to represent the radius of the pitch-cylinder A; then perpendicular to I C draw B D parallel to I A, cutting I N in D; then perpendicular to I N draw D R, cutting I C in R; I R will be the radius of curvature of the normal helix of the screw A. A similar construction, substituting small for capital letters, serves to find I r, the radius of curvature of the normal helix of the screw a.

Fig. 111 represents two projections of the pitch-cylinders of a pair of screws designed by the rules which have just been given, and shows also the helical lines in which the fronts of the threads cut those pitch-cylinders. The upper part of the figure is a projection on the plane of action, whose trace, in fig. 110, is I N. A′ a' is *the common perpendicular* of the two axes, and I the

[redacted]

pitch-point; N′N′ is the trace of the common tangent plane of the two pitch-cylinders; and the arrow shows the direction of the common component of their surface velocities at the point I′. R and *r* are the centres of curvature of the two normal screw-lines at the point I′; and S S and *s s*, described about R and *r* respectively, are their two osculating circles, whose radii, I′ R and I′ *r*, are found by Rule III.

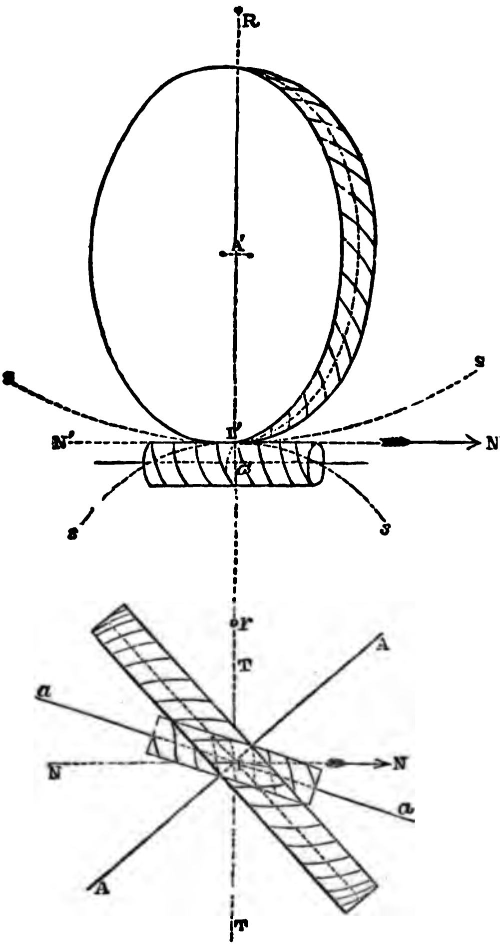

Fig. 111.

The lower part of the figure is a projection on the common tangent plane of the pitch-cylinders. A A and *a a* are the projections of their two axes; T I T is the line of contact; N I N is the trace of the plane of action; and the arrow marks the direction of the common component of the surface velocities at the pitch-point I.

In the particular example represented by figs. 109, 110, and 111, the following are the principal data and proportions:—

$$\text{Velocity-ratio } \frac{a}{A} = 20;$$

Number of threads of A, 40; of *a*, 2;

Ratios of radii and line of centres,

$$B + b : B : b$$
$$:: \ 11 \ : 10 \ : 1$$

Both screws right-handed.

155. **Figures of Threads found by Means of Normal Screw-Lines.**—By the following process threads may be designed for any gearing screw, so that they shall gear correctly with threads designed on the same principle for any other screw of the same normal pitch.

Let the screw to be provided with threads be, for example, the screw A of fig. 111. Draw, by Rule III. of Article 154, page 161, the osculating circle, S I′ S, of its normal screw-line. Lay off the normal pitch upon that osculating circle, and design the figure of a tooth and two half-spaces of that pitch, with the proper addendum and depth, as if the osculating circle were the pitch-circle of a spur-wheel; the figure so drawn will be the *normal section* of a thread, being the trace of the thread upon a surface which cuts it at right angles; and by the help of that section the threads may be made of the correct figure.

The normal sections of the acting surfaces of a thread may be either involutes of circles (Articles 131, 133, pages 120 to 128), or epicycloïds (Articles 136 to 140, pages 130 to 137). All screws with *involute threads* of the same divided normal pitch gear correctly together, and may be said to belong to *one set;* and they have the same property with involute toothed wheels, of admitting of some alteration of the distance between the axes. All screws of the same divided normal pitch having epicycloïdal teeth described by the same rolling circle gear correctly together, and may be said to belong to *one set.*

This method of designing the threads of gearing screws is believed to be now published for the first time.

156. **Figures of Threads designed on a Plane Normal to one Axis.**—In many cases which occur in practice the axes of the two screws are perpendicular to each other; so that, in fig. 110, page 160, A I P and *a* I *p* are at right angles, I C coincides with I *p*, and I *c* coincides with I P; and therefore the *divided axial pitch of either screw is equal to the circular pitch of the other*. In such cases, and especially where the diameters of the pitch-cylinders are very unequal, so that the larger screw is called a *worm-wheel*, and the smaller an *endless screw*, it is often convenient to design the traces of the threads on a plane normal to the axis of the worm-wheel, and traversing the axis of the endless screw; and then it is evident (as Mr. Willis appears to have been the first to show) that if the traces of the threads of the worm-wheel be made like those of a spur-wheel *of the same radius* and pitch, and those of the threads *of the screw like the traces of* the teeth of a rack suited to gear

[redacted]

with that spur-wheel, the worm-wheel and screw will gear correctly together.

Fig. 112 represents a worm-wheel and endless screw.

The lower part of the figure is a diagram drawn on the common tangent plane of the pitch-cylinders. I is the pitch-point; I C is

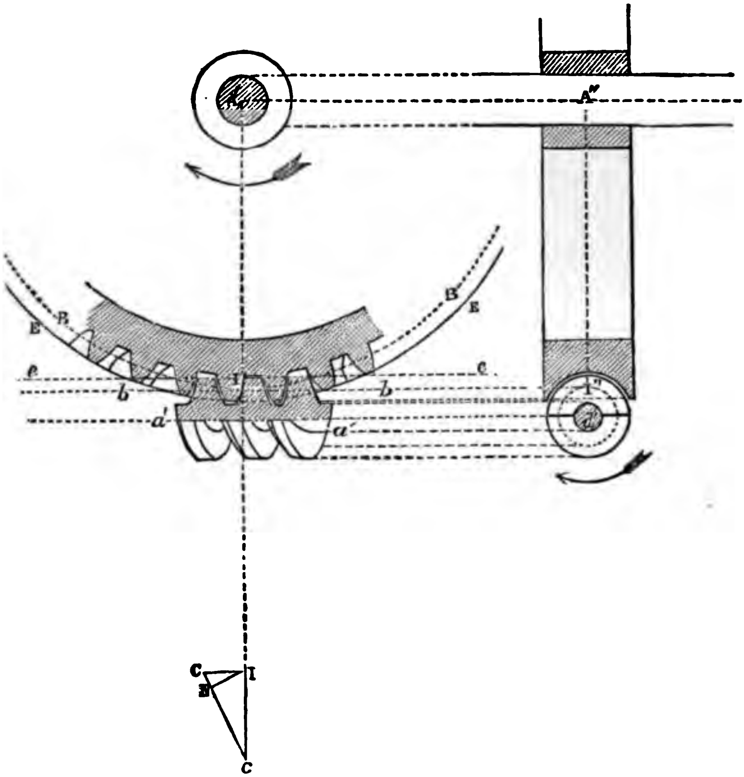

Fig. 112.

the divided axial pitch of the endless screw, being also the development of the circular pitch of the worm-wheel; I *c* is the divided axial pitch of the worm-wheel, being also the development of the circular pitch of the endless screw. I N, perpendicular to C *c*, is the development of the divided normal pitch of both screws; and *C c is the extent of* transverse sliding which takes place while an *arc equal to the pitch* passes the pitch-point.

In the left-hand division of the upper part of the figure the plane of projection is normal to the axis, A′, of the worm-wheel, and traverses the axis, a' a', of the endless screw. The circle B B is the trace of the pitch-cylinder of the wheel; the straight line b b is the trace of the upper side of the pitch-cylinder of the screw; and those traces touch each other in the pitch-point I′. The threads of the wheel, and those at the upper side of the screw, are shown in section; the traces of the threads of the wheel are like those of the teeth of a spur-wheel having the same circular pitch, and B B for a pitch-circle; the traces of the threads of the screw are like those of the teeth of a rack suited to gear with that spur-wheel, and having b b for its pitch-line. The addendum-circle, E E, of the worm-wheel, and the addendum-line, e e, of the endless screw, are drawn as for a spur-wheel and rack. The lower parts of the threads of the endless screw are shown in projection. In the example given, both wheel and screw have right-handed threads; the number of threads of the screw is two; of the wheel, 40; and the screw is represented as driving the wheel. The right-hand division of the upper part of the figure shows the wheel in section and the screw in projection; and the plane of projection traverses the axis, A″, of the wheel, and is normal to the axis, a'', of the screw; I″ is the pitch-point.

The traces of the threads of the wheel in the left-hand division of the upper part of the figure are involutes of a circle, and those of the threads of the screw are straight lines. That shape, as in the case of spur-wheels, enables the distance between the axes to be varied to a certain extent without affecting the accuracy of the action. But any shapes suited for the teeth of wheels and racks may be employed.

If a set of worm-wheels be made of the same circular pitch and obliquity of thread, and having the traces of the threads all involutes or all epicycloïds, traced by the same rolling circle; and if a set of endless screws be made, all of the same divided axial pitch, equal to the circular pitch of the wheels, and of an obliquity of thread equal to the complement of the obliquity of the threads of the wheels, and having the traces of the teeth, as the case may be, all straight lines of the proper obliquity, or all epicycloïds traced by the same rolling circle that is used to trace the threads of the wheels, then any one of the wheels will gear correctly with any one of the screws.

157. **Close-Fitting Tangent Screws.**—In many cases the object of screw-gearing is not the economical transmission of motive power, but the production of small angular motions with great accuracy: as, for example, when the principal wheel of a dividing engine, or that of a machine *for pitching* and cutting the teeth of wheels, or *the wheel or sector which* adjusts the direction of stroke of a

[redacted]

cutting tool in a shaping machine, is driven by a "tangent-screw" situated relatively to the wheel in the manner already shown in fig. 112. In such cases the screw has not only to move the wheel into any required position, but to hold it there; and therefore it is essential that there should be no back-lash. In order to ensure this, together with the requisite precision of action, an exact copy of the tangent screw is made of steel, the edges of its thread are notched, and it is hardened, so that it becomes a cutting tool: it is then mounted in a suitable frame, so as to gear with the roughly formed teeth or threads of the wheel, and turned so as to drive them; in the course of which operation it cuts them to the proper figure. The axis of the cutting screw is placed at first at a distance from the axis of the wheel somewhat greater than the intended permanent distance; and after each complete revolution of the wheel the axes are brought a little nearer together, until the permanent distance is attained; and by turning the screw in this last position the shaping of the teeth or wheel-threads is finished. From the property of threads with traces similar to those of involute teeth, which has already been mentioned in Article 156, page 165, it is evident that this class of figures is peculiarly well suited to cases in which the tangent-screw is made to cut the wheel, because of the gradual diminution of the distance between the axes which takes place during the process of cutting.

158. **Oldham's Coupling.**—A *coupling* is a mode of connecting a pair of shafts so that they shall rotate in the same direction, with the same mean angular velocity. If the axes of the shafts are in the same straight line, the coupling consists in so connecting their contiguous ends that they shall rotate as one piece; but if the axes are not in the same straight line, combinations of mechanism are required. Various sorts of couplings will be described and compared together in a later division of this treatise. The present Article relates to a coupling for parallel shafts, invented by Oldham, which acts by *sliding contact*. It is represented in fig. 113. C_1, C_2 are the axes of the two parallel shafts; D_1, D_2, two discs facing each other, fixed on the ends of the two shafts respectively; E_1 E_1, a bar sliding in a diametral groove in the face of D_1; E_2 E_2, a bar sliding in a diametral groove in the face of D_2: those bars are fixed together at A, at right angles to each other, so as to form a rigid cross. The angular *velocities of the two* discs and of the cross are all equal at every *instant; the middle* point of the cross, at A, revolves in the dotted

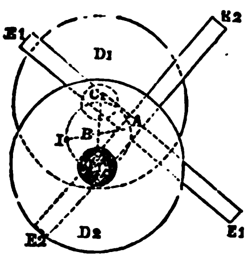

Fig. 113.

circle described upon the line of centres, C_1 C_2, as a diameter, twice for each turn of the discs and cross; the instantaneous axis of rotation of the cross at any instant is at I, the point in the circle C_1 C_2 diametrically opposite to A; and each arm of the cross slides in its groove through a distance equal to *twice the line of centres* during each half revolution, or twice the line of centres and back again—that is, four times the line of centres—during each revolution.

Oldham's coupling belongs to Mr. Willis's Class A. The cross may be strengthened by making its two bars take the form of projecting diametral ridges on opposite sides of a third circular disc Or the cross may consist of two grooves in the opposite sides of such a disc, and instead of grooved discs, the two shafts may carry cross bars fitting the grooves of the cross.

159. **Pin and Straight Slot.**—The communication of a uniform velocity-ratio by the sliding contact of a round pin with the sides of a slot or groove has already been described in Article 141, page 137. A velocity-ratio varying in any manner may be communicated by making the slot of a suitable figure, the principle of the combination being, that the line of connection is a normal to the centre line of the slot, traversing the centre line of the pin. The present Article relates to cases in which the slot is straight and the velocity-ratio variable. Three such cases are illustrated by figs. 114, 115, and 116, further on. Fig. 114 represents a *coupling*, belonging to Mr. Willis's Class B, where two shafts turn about the parallel axes A and B with equal mean angular velocities, though the angular velocity-ratio at each instant is variable. Fig. 115 shows a crank turning continuously about the axis A, and carrying a pin, C, which, by means of the slot F G, drives a lever which rocks or oscillates about the axis B. Fig. 116 shows a crank turning continuously about the axis A, and carrying a pin, C, which, by means of the slot F G in the cross-head of the rod B, gives a reciprocating sliding motion to that rod. The last two combinations belong to Mr. Willis's Class C.

In practice, for the purpose of diminishing friction and preventing back-lash, it is usual to make the pin turn in a bush which slides in the slot; but that bush is not shown in the figures.

The following are the principles of the action of those three combinations:—

I. *Coupling* (fig. 114).—In order that the directional relation of the rotations may be constant, the *crank-arm*, A C, must be greater than the line of centres, A B.

With a given crank-arm, A C, to find *the position of the axis* B *of the slot-lever*, so that the crank and slot-lever shall alternately overtake and fall behind each other by a given angle:—With the radius A C *describe the circle* D C E, and draw the diameter *D A E, with which the line* of centres is to coincide. Lay off

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E A H = E A h = the complement of the given angle, and draw H B h perpendicular to D A E. B will be the trace of the required axis.

At the instant when the centre of the pin is at H or h, the angular velocities are equal; and A H B = A h B is the given angle beforementioned.

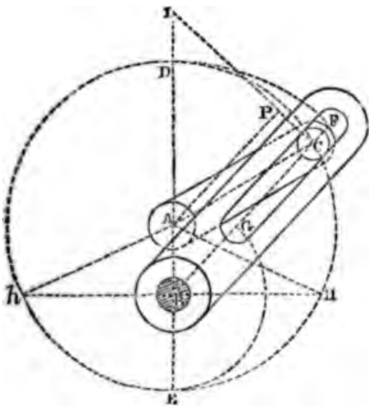

Fig. 114.

With a given position, C, of the centre of the pin, to find the *angular velocity-ratio*:—From C, perpendicular to the centre line, B C, of the slot, draw the *line* of connection, C I, cutting the line of centres in I; then

$$\frac{\text{Angular velocity of B}}{\text{Angular velocity of A}} = \frac{\text{A I}}{\text{B I}};$$

or otherwise: draw A P parallel to B C and perpendicular to C I; then

$$\frac{\text{Angular velocity of B}}{\text{Angular velocity of A}} = \frac{\text{A P}}{\text{B C}}.$$

The $\left\{\begin{matrix}\text{greatest}\\ \text{least}\end{matrix}\right\}$ values of this ratio occur when the pin is at $\left\{\begin{matrix}\text{E}\\ \text{D}\end{matrix}\right\}$ respectively; and they are as follows:—

$$\text{Greatest, } \frac{\text{A E}}{\text{B E}} = \frac{\text{A C}}{\text{A C} - \text{A B}};$$

$$\text{Least, } \frac{\text{A D}}{\text{B D}} = \frac{\text{A C}}{\text{A C} + \text{A B}}.$$

The *travel* or *length of sliding of the pin in the slot* is

$$\text{F G} = \text{B F} - \text{B G} = \text{B D} - \text{B E};$$

and this takes place twice in each revolution.

II. *Crank and Slotted Lever* (fig. 115).—As the crank-arm, A C, in fig. 115, is shorter than the line of centres, A B, the slotted lever, B G F, has a reciprocating or rocking motion.

With a given line of centres, A B, and a given *semi-amplitude* or angular half-stroke of the rocking motion of the lever, A B K = A B k, to find *the length of crank-arm*:—From A let fall A K perpendicular to B K, or A k perpendicular to B k; A K = A k *will be the* required crank-arm.

K and k will be the two *dead points*; that is to say, the positions

of the centre of the pin at the two instants when the lever has no velocity, having just ceased to move in one direction, and being just about to begin to move in the opposite direction.

To find the *angular velocity-ratio* at the instant when the centre of the pin is in a given position, C:—Draw the corresponding position, B C F, of the centre line of the slot, and perpendicular to it draw C I, cutting the line of centres in I; then

$$\frac{\text{Angular velocity of lever}}{\text{Angular velocity of crank}} = \frac{\text{A I}}{\text{B I}}.$$

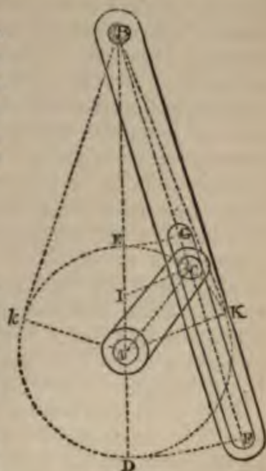

Fig. 115.

To find the travel of the pin in the slot, lay off B G = B E, and B F = B D; G and F will be the two ends of the travel of the centre of the pin; and F G = D E = 2 A C will be the length of travel.

III. *Crank and Slot-headed Sliding Rod* (fig. 116).—The crank-arm, A C, in this case is to be made equal to one-half of the intended *length of stroke* of the sliding rod, B. Draw the circle described by C, the centre of the pin, and let k A K be the diameter of that circle which is parallel to the direction of motion of the rod; then K and k will be the *dead points*, or positions of the centre of the pin at the two instants when the rod has no velocity. To find the *velocity-ratio* of the rod and crank-pin when the centre of the crank-pin is in a given position, C: perpendicular to the direction of motion of the rod draw the diameter D A E; this line will correspond to the line of centres in the preceding problems; then through C, and perpendicular to the centre-line, F G, of the slot, draw the line of connection, C I, cutting D A E in I; the following will be the required velocity-ratio:—

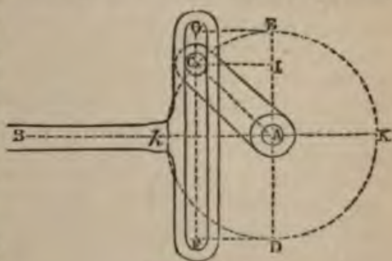

Fig. 116.

$$\frac{\textit{Velocity of rod, B}}{\textit{Velocity of centre of pin, C}} = \frac{\text{A I}}{\text{A C}}.$$

[redacted]

The *extent of travel of the pin in the slot* is F G = D E = 2 A C.

160. **Cams and Wipers in General.**—Cams and wipers are those primary pieces, with curved acting surfaces, which work in sliding contact without being related to imaginary pitch-surfaces, as the teeth of wheels and threads of screws are. The distinction between a cam and a wiper is, that a cam in most cases is continuous in its action, and a wiper is always intermittent; but a wiper is sometimes called a cam notwithstanding. A cam is often like a noncircular sector or wheel in appearance; a wiper is often like a solitary tooth. (As to "rolling cams," see Article 110, page 99.)

The solutions of all problems respecting the velocity-ratio and directional relation in the action of cams and wipers are obtained by properly applying the general principle of Article 122, page 114.

In most cases which occur in practice, the condition to be fulfilled in designing a cam or a wiper does not directly involve the velocity-ratio, but assigns a certain series of definite positions which the follower is to assume when the driver is in a corresponding series of definite positions. Examples of such problems will be given in the following Articles.

161. **Cam with Groove and Pin.**—Throughout the present Article it will be supposed that the acting surface of the follower, which is to be driven by the cam, is the cylindrical surface of a pin. It is easy to see that without in any respect altering the action, a cylindrical roller turning about a smaller pin may be substituted for a pin in order to diminish friction. If the pin is to be driven by the cam in one direction only, being made to return at the proper time by the force of gravity or by the elasticity of a spring, the cam may have only one acting edge; but if the pin is to be driven back as well as forward by the cam, the cam must have two acting edges, with the pin between them, so as to form a groove or a slot of a uniform width equal to the diameter of the pin, with clearance just sufficient to prevent jamming or undue friction. The centre of the pin may be treated as practically coinciding at all times with the centre-line of such a groove, which centre-line may be called the *pitch-line* of the cam. The most convenient way to design a cam is usually to draw, in the first place, its pitch-line, and then to lay off the half-breadth of the groove on both sides of the pitch-line. When one acting edge only is required, it is to be laid off on one side of a groove, the other side being omitted.

The *line of connection* at any instant is a straight line normal to the pitch-line at the centre of the pin.

The surface in which the groove is made may be either a plane or a surface of revolution; a plane for a *cam-plate* which either turns about an axis normal to its own plane or slides in a straight line, and acts upon a pin whose centre moves in a plane parallel to that of the cam-plate; a solid of revolution, being either a cylinder,

a cone, or a hyperboloïd, for a cam which turns about an axis, and acts on a pin whose centre has a reciprocating motion in a straight line coinciding with a generating line of the surface of revolution.

The following example is a case of a rotating plane cam, giving motion through a pin and lever to a rocking shaft whose axis is parallel to the axis of rotation of the cam.

In fig. 117 the plane of projection is that of the cam-plate, and is normal to the axes of the cam and of the lever. In the lower part of the figure, A′ represents the trace of the axis of the rocking

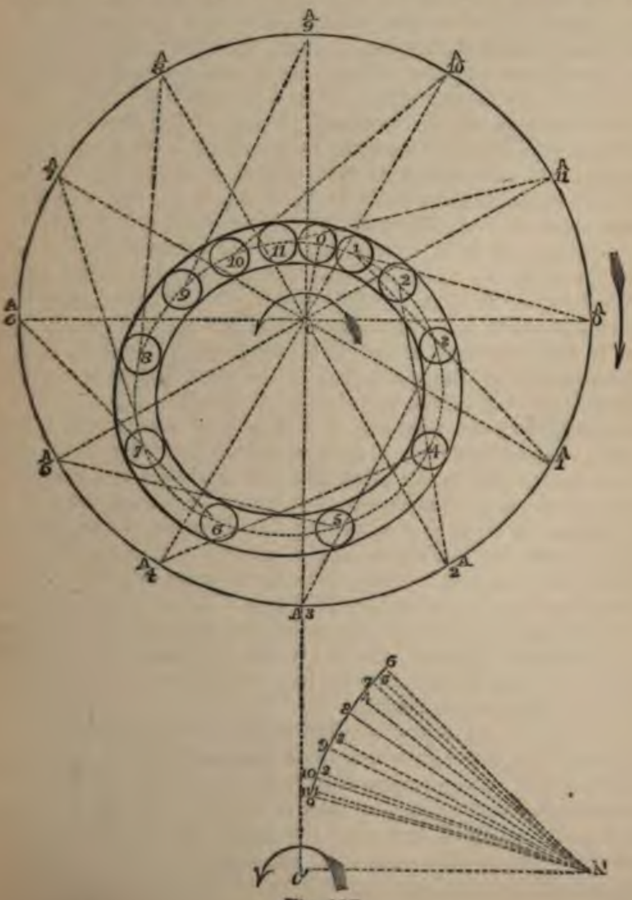

Fig. 117.

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shaft, and C′ the trace of the axis of the cam, so that A′ C′ is the line of centres. The direction of rotation of the cam is shown by an arrow. In the example, the direction is left-handed. The circular arc, 0 6, described about A′ with the radius A′ 0, is the path to be described by the centre of the pin; and the twelve points in that arc, marked with numbers from 0 to 11, are twelve positions which the centre of the pin is to occupy at the end of twelve equal divisions of a revolution of the cam. It is required to find the form of the cam which will produce that motion in the pin.

In the upper part of the figure, let C represent the axis of the cam; suppose that the cam is fixed, and that the line of centres, C A, rotates about C, carrying the axis, A, of the rocking shaft along with it, with an angular velocity equal and contrary to the actual angular velocity of the cam. That supposition will not alter the relative motions of the working pieces. With the radius C A describe a circle to represent the supposed path of A relatively to C; divide its circumference into twelve equal parts, and to the points of division draw radii, C A_0, C A_1, C A_2, &c., to represent twelve successive positions of the line of centres relatively to the cam, as supposed to be fixed. Lay off the angles C A_0 0, C A_1 1, C A_2 2, &c., in the upper part of the figure respectively, equal to the angles C′ A′ 0, C′ A′ 1, C′ A′ 2, &c., in the lower part of the figure; and make each of the straight lines A_0 0, A_1 1, A_2 2, &c., equal to the lever arm A′ 0. The points thus found, 0, 1, 2, &c., will be points in the pitch-line of the cam, and a curve drawn through them will be the required pitch-line.

About each of the points 0, 1, 2, &c., draw a circle of a radius equal to that of the pin: a pair of curves touching those circles so as to be parallel to the pitch-line will mark the two sides of the groove, without allowance for clearance. Clearance may be provided either by slightly diminishing the diameter of the pin or by slightly increasing the width of the groove. If the lever is to be raised by the cam, but brought down again by gravity, the outer side of the groove may be omitted, and the cam will become a disc bounded by the innermost of the three parallel curves shown in the figure.

The number of parts into which the revolution of the cam is divided may be made more or less numerous according to the degree of precision required.

It is easy to see how a similar method may be applied to the designing of a cam-disc which shall produce a given motion in a follower whose acting surface is of any given form. A figure is to be constructed like the upper part of fig. 117, on the supposition that the cam is fixed, and that the frame of the machine rotates about the axis of the cam with an angular velocity equal and contrary to the actual angular velocity of the cam. Then, just as the pin in the upper part of fig. 117 is drawn in its several positions,

[redacted]

0, 1, 2, &c., the trace of the acting surface of the follower is to be drawn in its several successive positions; and a line touching that trace in all its positions will be the trace of the required cam-disc.

The *dead points* of a cam are the points in its pitch-line which are at the greatest and least distances from its axis. In the example shown in fig. 117 the dead points are 0 and 6. When the centre of the pin is at those points it has no velocity. Any part of the pitch-line which is an arc of a circle about C corresponds to a *pause* in the motion of the pin.

162. **Drawing a Cam by Circular Arcs.**—In many cases in which cams have to be designed, the dead points alone are given by the conditions of the problem, leaving the parts of the pitch-line between those points to be drawn according to convenience. For example, in fig. 118,

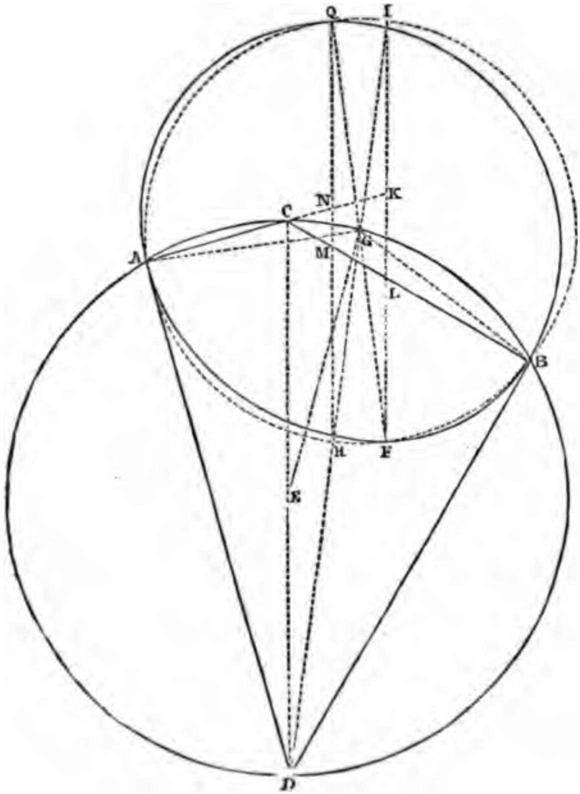

Fig. 118.

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